Image Processing
Unit-2

Section 2.3 to 2.6 in the text book*

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**Digital Image Processing by Rafael Gonzalez and Richard Woods*

Topics in this unit

- Image Sensing and Acquisition
- Image Sampling and Quantization
- Some Basic Relationships
- Nonlinear Operations

Image Sensing and Acquisition

- Image Acquisition Using a Single Sensor
- Image Acquisition Using Sensor Strips
- Image Acquisition Using Sensor Arrays
- A Simple Image Formation Model

Image Sampling and Quantization

- Basic Concepts in Sampling and Quantization
- Representing Digital Images
- Spatial and Gray-Level Resolution
- Aliasing and Moiré Patterns
- Zooming and Shrinking Digital Images

Basic Relationships Between Pixels

- Neighbors of a Pixel
- Adjacency, Connectivity, Regions, and Boundaries
- Distance Measures
- Image Operations on a Pixel Basis
- Linear and Nonlinear Operations

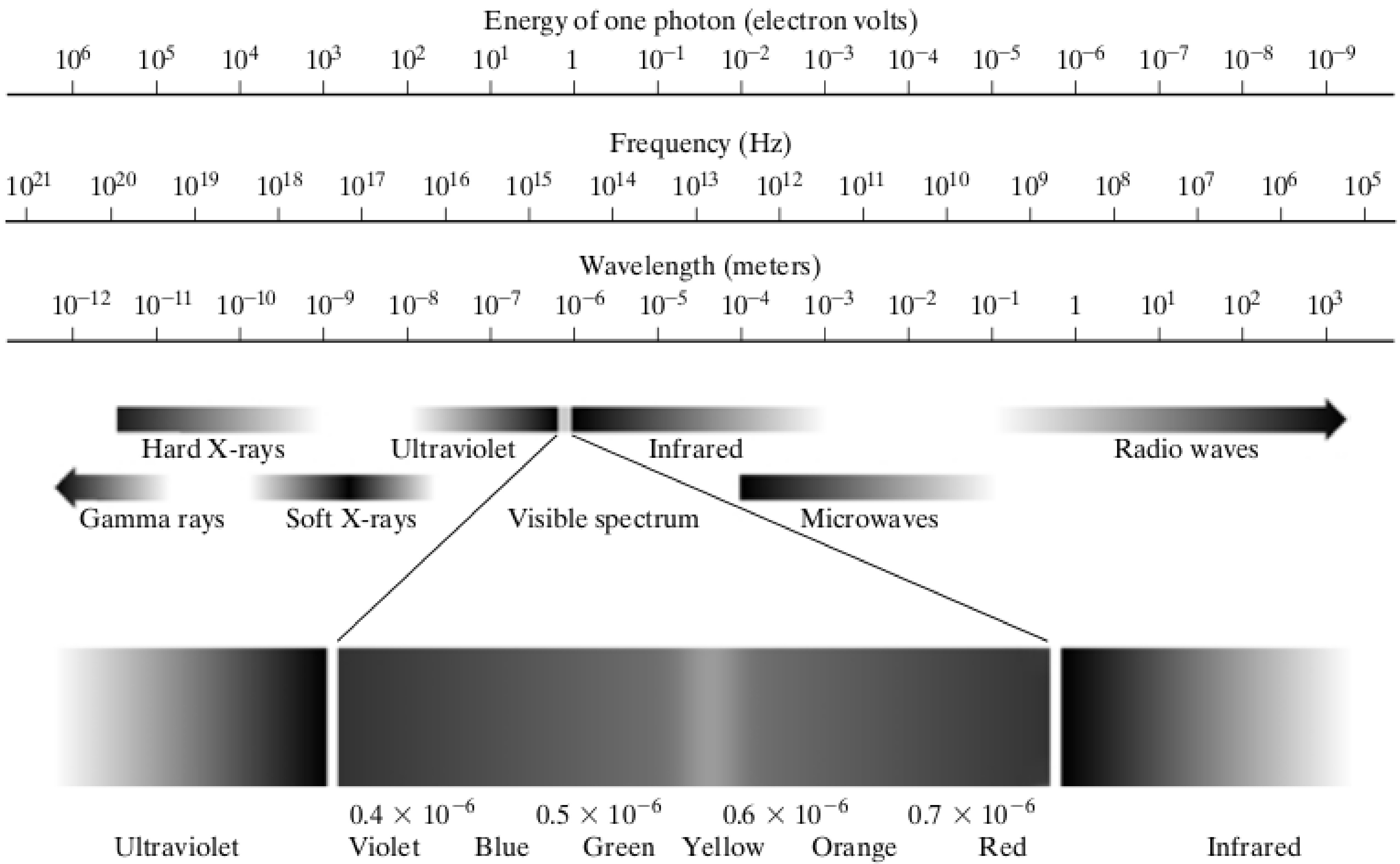


FIGURE 2.10 The electromagnetic spectrum. The visible spectrum is shown zoomed to facilitate explanation, but note that the visible spectrum is a rather narrow portion of the EM spectrum.

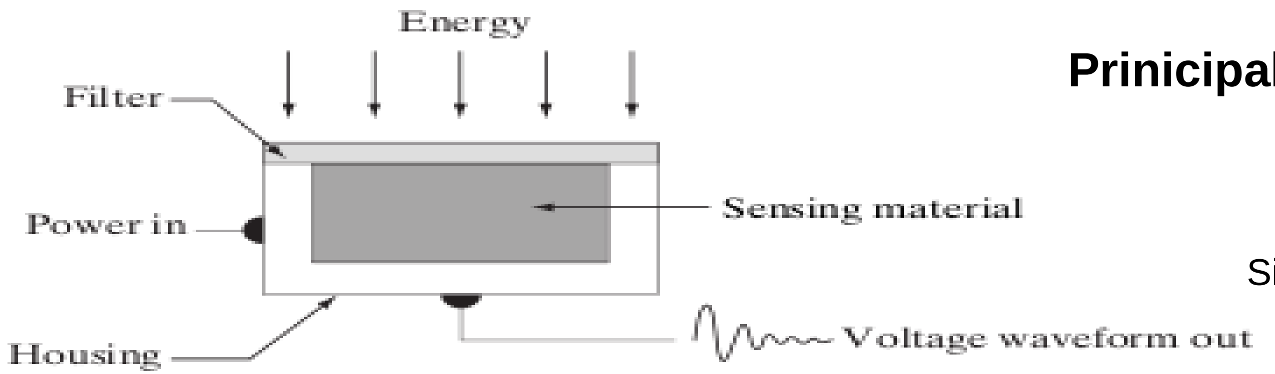
Image Sensing and Acquistion

Images are generated by the combination of an “illumination” source and the reflection or absorption of energy from that source by the elements of the “scene” being imaged.

Illumination : EM waves, ultrasound, computer generated image pattern ....

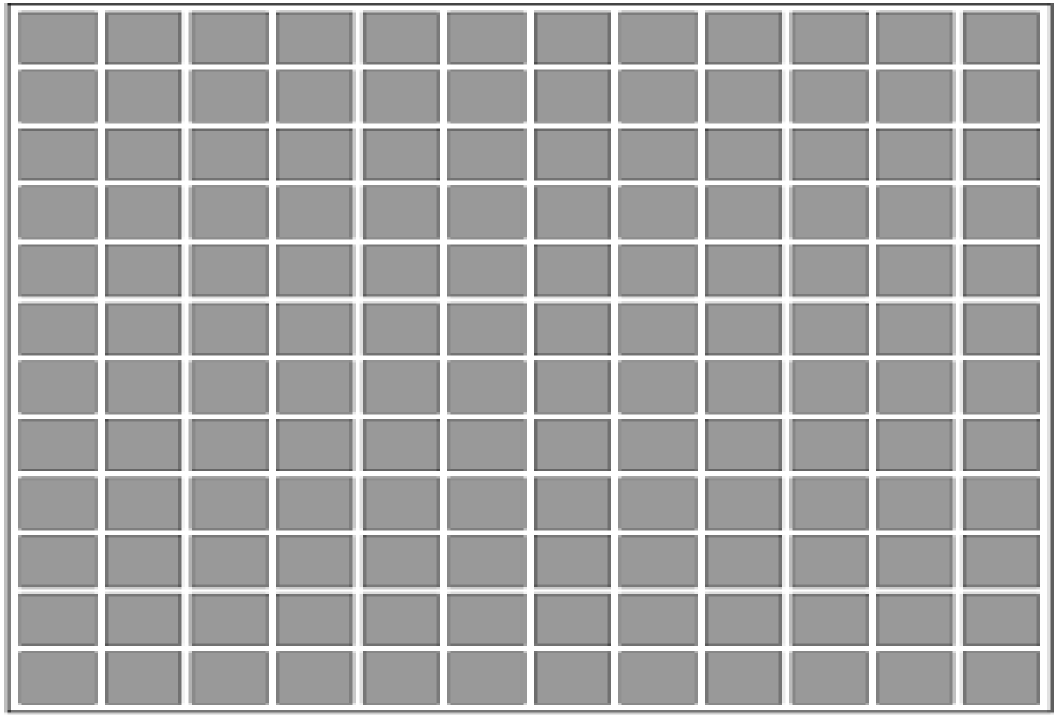
Scene: Molecules, buried rock formations, human brain ...

Prinicipal sensor arrangements



Single imaging sensor

Linear sensor

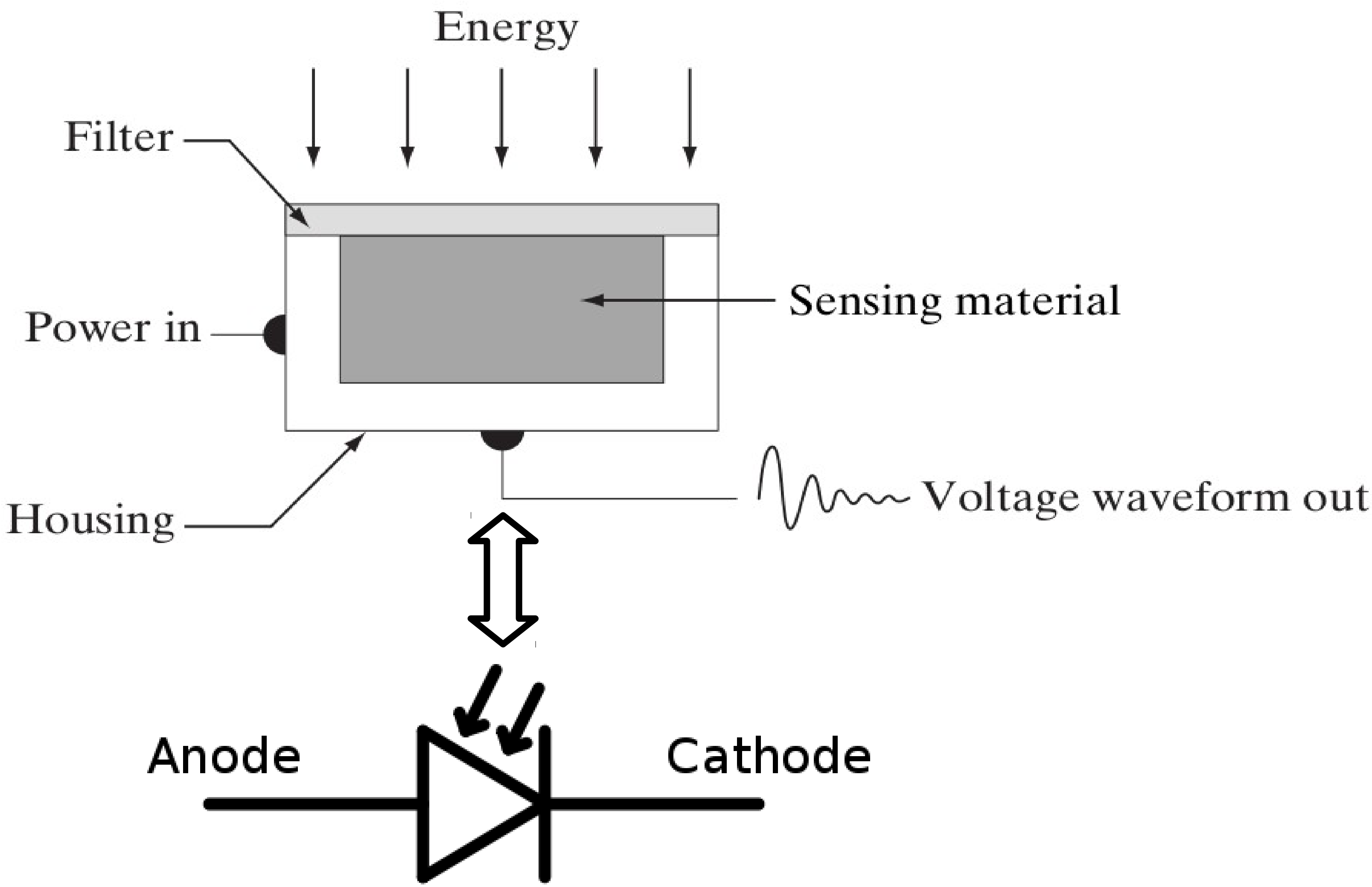


Array sensor

Principle behind image sensing

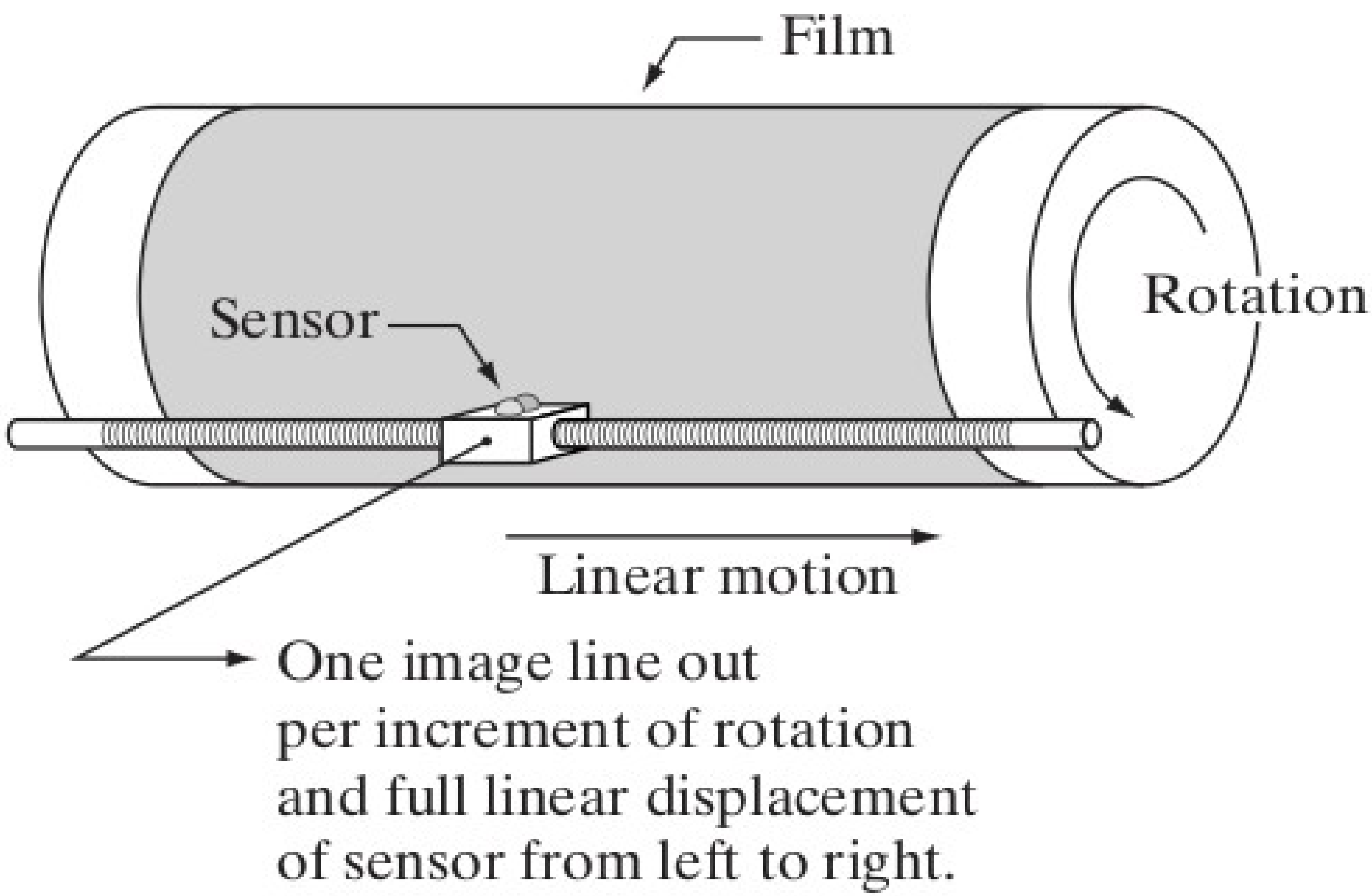
- Incoming energy is transformed into a voltage
- Combination of *input electrical power* and *sensor material* that is responsive to the particular type of energy being detected
- The output voltage waveform is the response of the sensor(s)
- A digital quantity is obtained from each sensor by digitizing its response

Image Acquisition Using a Single Sensor



Photodiode

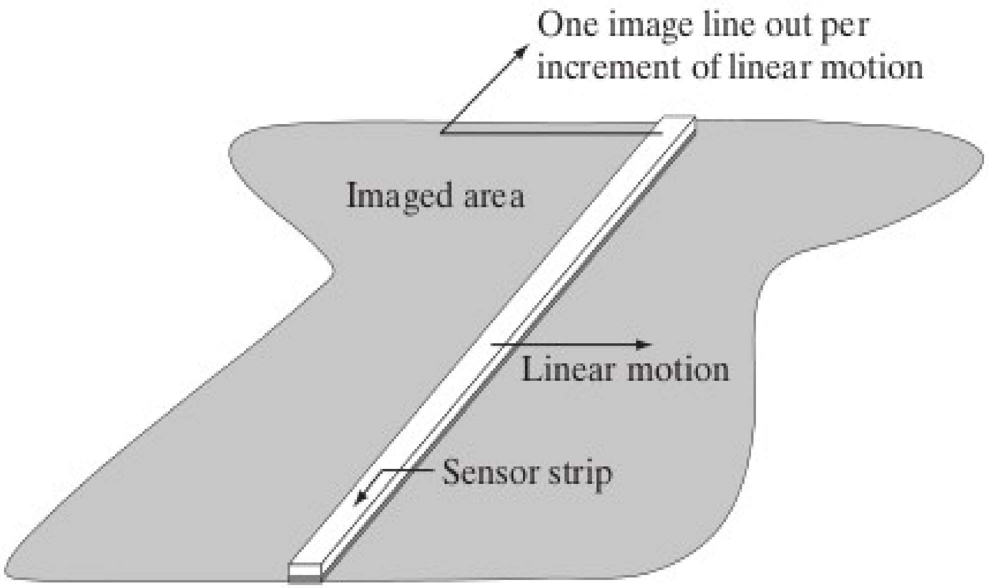
2-D image using a single sensor



2-D image using a single sensor

- There has to be relative displacements in both the x- and y-directions between the sensor and the area to be imaged.
- A film negative is mounted onto a drum whose mechanical rotation provides displacement in one dimension
- The single sensor is mounted on a lead screw that provides motion in the perpendicular direction.
- Used in high-precision scanning
- **Adv:** Mechanical motion can be controlled with high precision, this method is an inexpensive
- **Disadv:** It is a slow method to obtain high-res images

Image Acquisition Using Sensor Strips



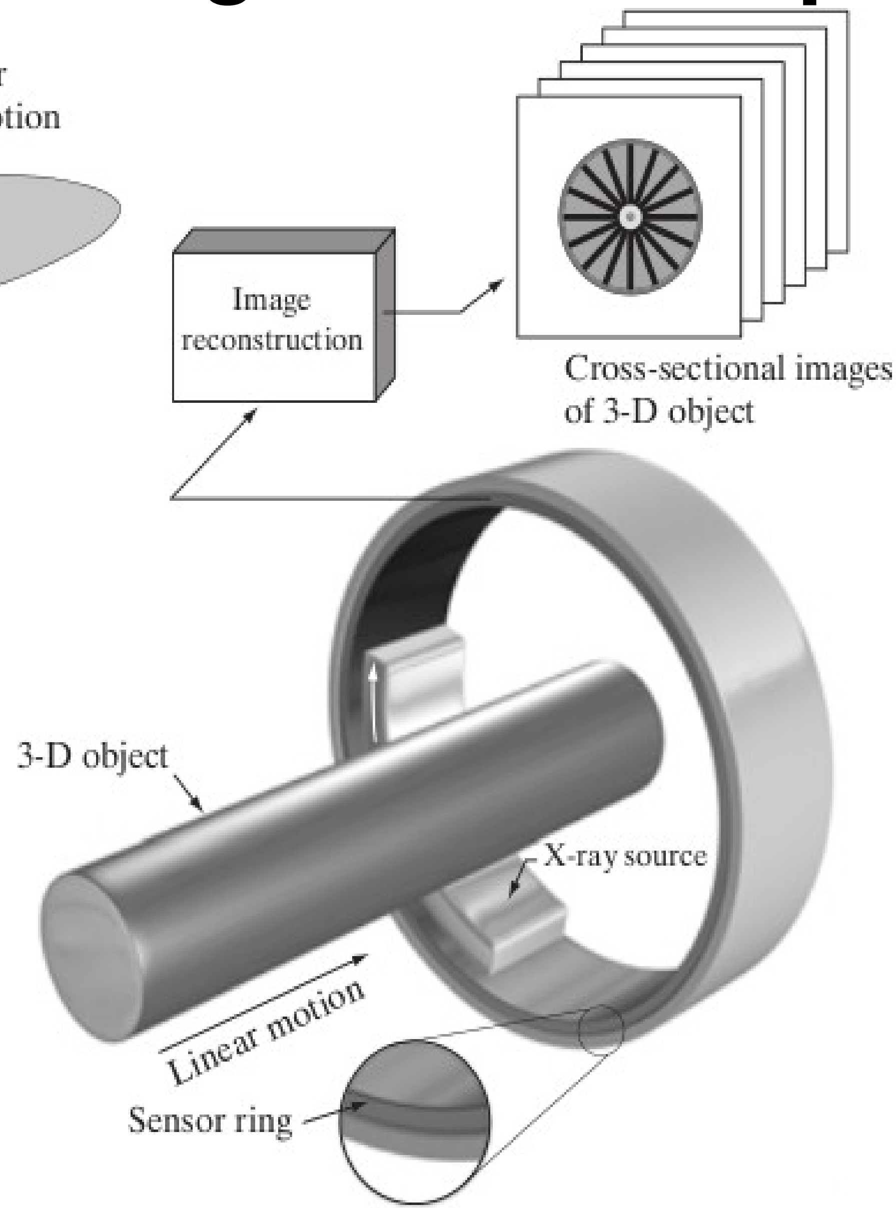


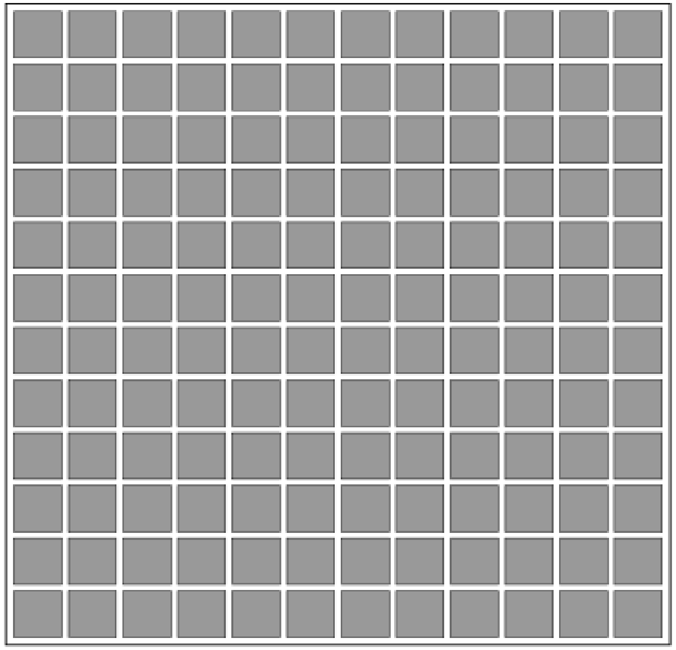
Image Acquisition Using Sensor Strips

- More frequently than single sensors consists of an in-line arrangement of sensors in the form of a *sensor strip*
- The strip provides imaging elements in one direction.
- Motion perpendicular to the strip provides imaging in the other direction
- Sensing devices with 4000 or more in-line sensors are possible.
- Used in most flat bed scanners, airborne applications

Image Acquisition Using Sensor Strips mounted in ring configuration

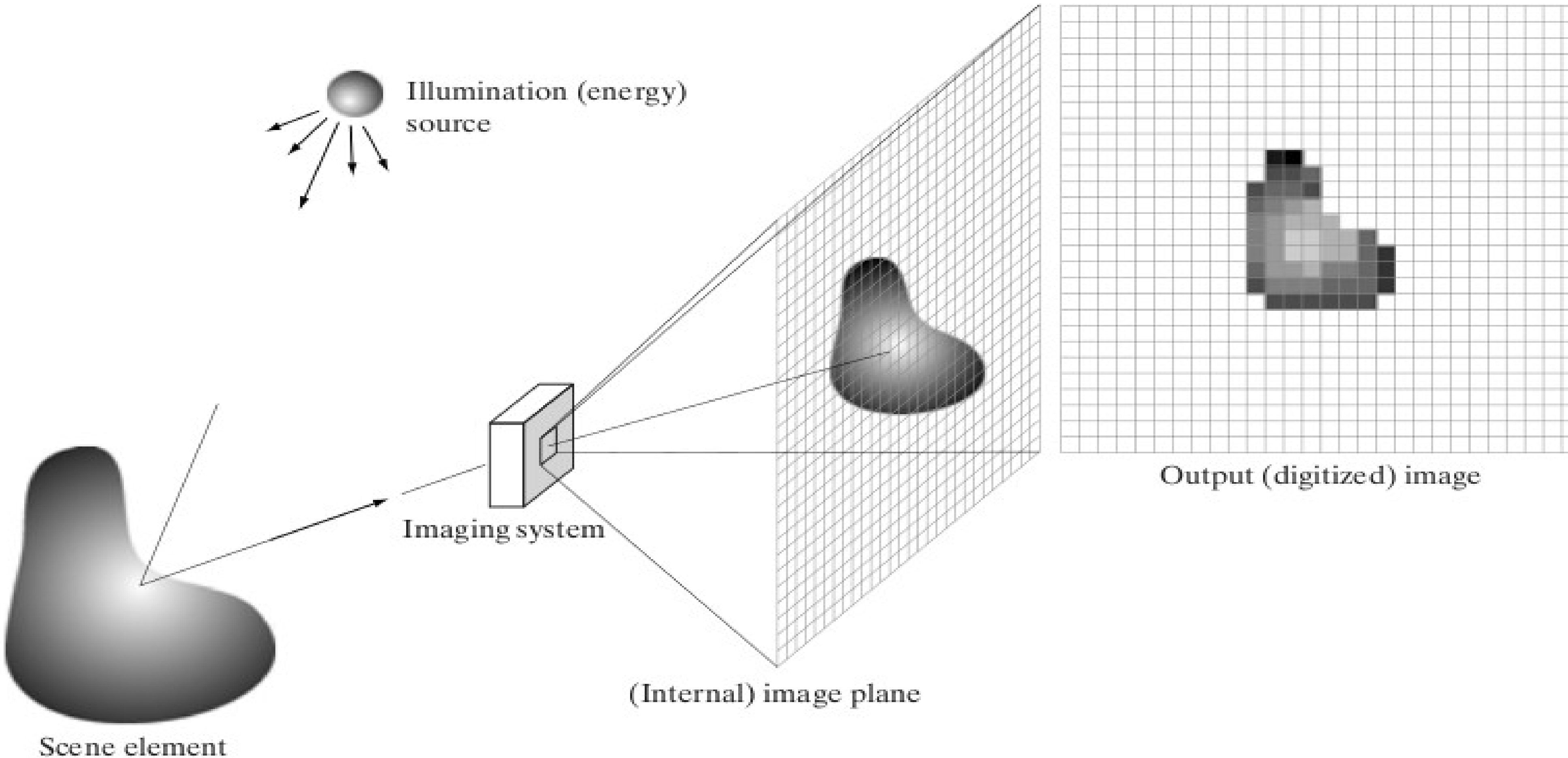
- Sensor strips mounted in a *ring configuration*
- A rotating X-ray/ similar source provides illumination
- Potion of the sensors opposite the source collect the X-ray energy that pass through the object
- Basis for medical and industrial computerized axial tomography (CAT)
- Output of the sensors must be processed by reconstruction algorithms whose objective is to transform the sensed data into meaningful cross-sectional images.
-
- Used in medical and industrial imaging to obtain cross-sectional ("slice") images of 3-D objects
- Magnetic resonance imaging (MRI) and positron emission tomography (PET).

Image Acquisition Using Sensor Arrays

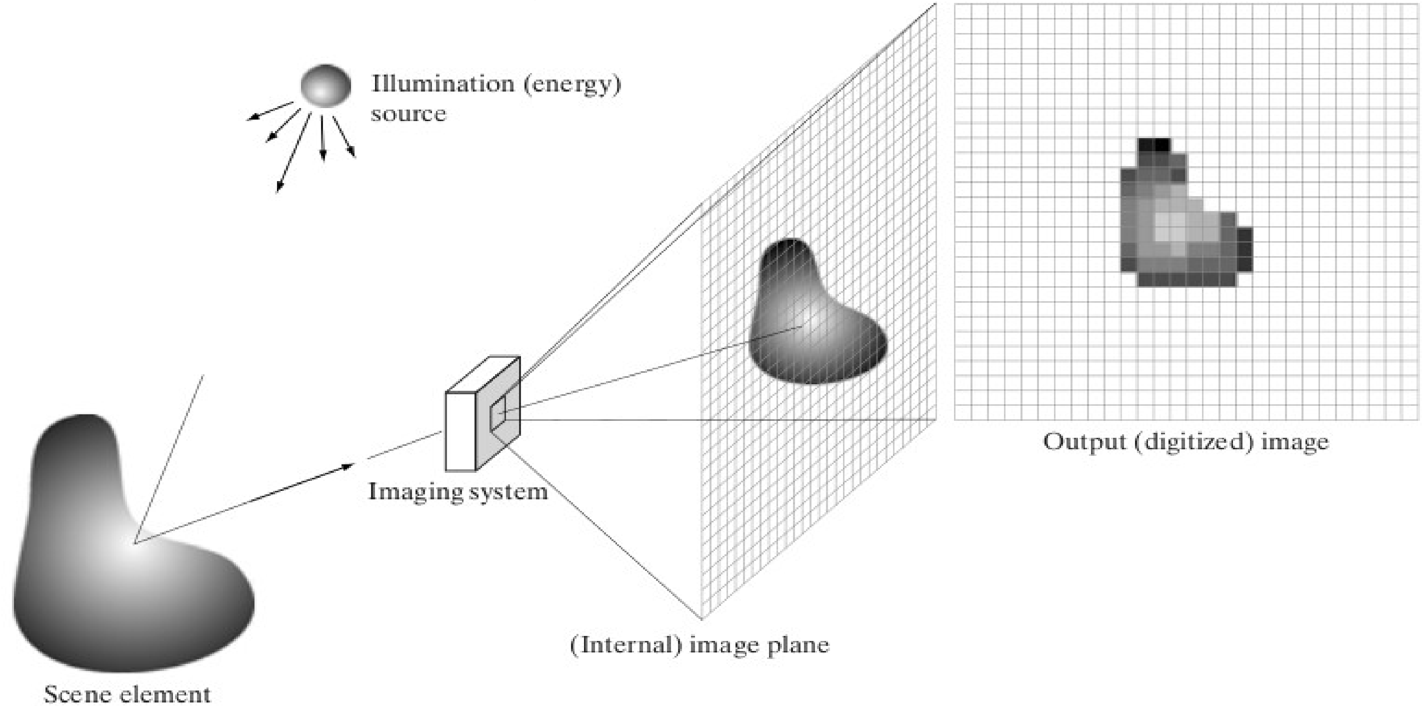


- Individual sensors arranged in the form of a 2-D array.
- Packaged in rugged arrays of 4000 * 4000 elements or more
- CCD sensors are used widely in digital cameras and other light sensing instruments
- The response of each sensor is proportional to the integral of the light energy projected onto the surface of the sensor
- Noise reduction is achieved by letting the sensor integrate the input light signal over minutes or even hours
- Since array is 2D, complete image can be obtained and hence motion of elements is not necessary

Principle behind Image Acquisition Using Sensor Arrays



Principle behind Image Acquisition Using Sensor Arrays



- Collects the incoming energy and focus it onto an image plane
- Projects the viewed scene onto the lens focal plane, if the illumination is light
- The sensor array, which is coincident with the focal plane, produces outputs proportional to the integral of the light received at each sensor
- Digital and analog circuitry sweep these outputs and convert them to a video signal, which is then digitized by another section of the imaging system.
- Yiedling the output, that is a digital image

A Simple Image Formation Model

Call me GNU!

A Simple Image Formation Model

```
octave:14> x=imread('gnu.jpg')
x =

  255  255  251  255  252  251  219  203  206  228  253  255  254  255  253  255  255  250  255  255  249  255  255  249  238  243  248  255  252  255  254  252
  250  251  255  222  167  182  167  178  137  139  155  208  249  244  255  255  253  255  252  253  255  237  175  168  163  175  188  179  230  254  245  255
  255  253  195  174  183   53   73  128  157  140  187  209  133  224  250  253  251  254  255  255  156  191  191  139  128  105   63  139  184  191  255  251
  255  191  186  119   16  170  165  184  208  199  181  190  204  255  255  254  255  254  255  249  231  185  192  188  201  180  174   75   60  186  213  255
  250  165  195    0  185  167  255  247  254  255  252  238  201  186  203  238  254  255  234  198  207  236  242  255  255  255  221  191   54  132  168  248
  195  240   69   69  178  252  248  255  253  253  209  182  218  173  175  178  203  176  180  182  190  205  195  251  254  255  251  191  137   44  218  209
  183  237   55   97  176  255  253  255  255  212  186  113   10    0    6   52  169  110  108   30    3   51  183  202  255  246  255  254  135   22  222  189
  175  245   37  114  203  253  255  251  219  173   80    3    0    3    0    3   79  251  181  158   22    2   33  171  214  255  252  255  146   21  231  170
  182  236   56  118  180  255  255  230  170   95    1    4   14   32   16   82  134  252  226  178   69    4    1   61  173  240  254  242  143   31  227  177
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  194  223  113    4  142  181  155   72  113  243  255  240  135   42  233  250  166  214  186  195   87  187  235  245   60   61  162  164    7   79  216  207
  251  176  184   16    4   34   22   80  255  254  247  176  213  200  222  198  219  199  247  211  121  220  220  252  201    7    6    6   16  168  171  244
  253  197  203  175   43    8    0  208  249  255  250  245  249  215  118   77  183  221  246  228   85   59  212  255  255   48   18   48  177  198  216  255
  255  255  183  189  229  146  102  252  255  251  253  216  236  229  151   79  225  253  255  244  130   90  227  250  255  182  107  232  180  194  246  255
  255  255  253  216  153  157  164  255  255  255  255  170  247  250  251  225  221  250  249  255  241  153  176  255  255  249  143  183  237  255  255  249
  253  253  255  250  255  217  196  255  252  255  222  111  255  254  249  255  254  255  254  253  255  195  134  166  206  255  216  155  252  255  247  255
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  254  249  255  254  226  157  255  219  163   87  248  255  247  251  255  253  140  150  201  230  182  227  247  220  164  250  255  255  255  250  254  255
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  255  255  250  251  190  210  216  149  251  155  231  248  255  255  243  207  230  248  251  236  246  252  255  223  155  254  250  247  255  251  255  254
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  254  255  255  255  255  249  252  129  255  251  253  111  223  255  250  251  239  165  164  171  166  129   95  136  253  247  255  255  250  255  255  255
  255  249  255  254  252  255  255  162  227  255  250  145  221  246  255  254  255  249  243  246  248  255  203  226  255  249  255  250  255  255  255  253
  255  255  253  253  255  255  249  241  163  252  255  235  121  129  253  255  255  242  182  125   98  131  163  247  255  255  255  255  255  255  255  255
  255  255  255  254  255  255  254  250  194  206  250  255  237  105  153  202  248  255  209  207  130  252  245  255  255  255  255  255  255  255  255  255
  254  255  255  255  254  255  255  255  253  194  232  255  233   93   35   63  253  253  255  237  197  213  255  254  255  255  255  255  255  255  255  255
  254  255  255  255  253  253  254  255  255  251  211  245  255  232   83   66  172  254  213  117  142  249  250  250  255  255  255  255  255  255  255  255
  255  255  255  255  255  254  253  253  255  254  255  252  250  234  111  193   18  150   17   10  220  252  255  255  255  255  255  255  255  255  255  255
  255  254  254  255  255  255  255  254  255  252  252  254  255  238  213  169   38   40    4    0   38  140  241  250  255  255  255  255  255  255  255  255
  255  253  254  255  255  255  255  255  251  255  254  254  255  251  255  223  191   38   33   62   27  187  243  255  255  255  255  255  255  255  255  255
  255  254  254  255  255  253  254  255  255  253  255  255  255  255  250  255  252  237  234  229  244  255  255  250  255  255  255  255  255  255  255  255
```

Array reading the pixel values of the 32x32 gnu image

http://ece756.wikispaces.com/

A Simple Image Formation Model

- Digital images by two-dimensional functions of the form

$$f(x, y)$$

- The value or amplitude of f at spatial coordinates (x, y) is a positive scalar quantity
- When an image is generated from a physical process, its values are proportional to energy radiated by a physical source
- $f(x, y)$ must be nonzero and finite i.e,

$$0 < f(x,y) < \infty$$

A Simple Image Formation Model

f(x, y) may be characterized by two components:

(1) The amount of source illumination incident on the scene being viewed – *Illumination component* - ***i(x, y)****
(2) the amount of illumination reflected by the objects in the scene – *Reflectance component* – ***r(x,y)*****

$$f(x, y)=i(x, y)r(x, y)$$

*** $0 < i(x,y) < \infty$**
**** $0 < r(x,y) < \infty$**

A Simple Image Formation Model

Gray Level (l)

$$l = f(x_0, y_0)$$

$$L_{min} \le l \le L_{max}$$

In theory, the only requirement on L_{min} *is that it be positive, and on* L_{max} *that it be finite.*

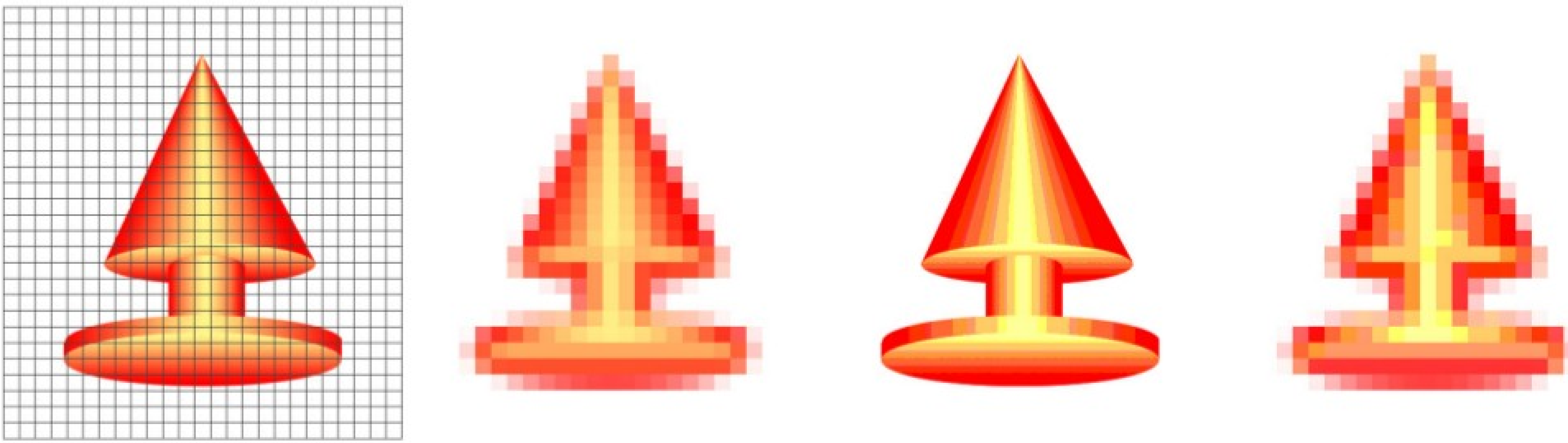
In practice, $L_{min} = i_{min} * r_{min}$ *and* $L_{max} = i_{max} * r_{max}$

A Simple Image Formation Model

Gray Level (l)

- The interval [L_{min} , L_{max}] is called the gray scale.
- This interval takes values [0, L-1], where l=0 is considered black and l=(L-1) is considered white on the gray scale.
- All intermediate values are shades of gray varying from black to white.

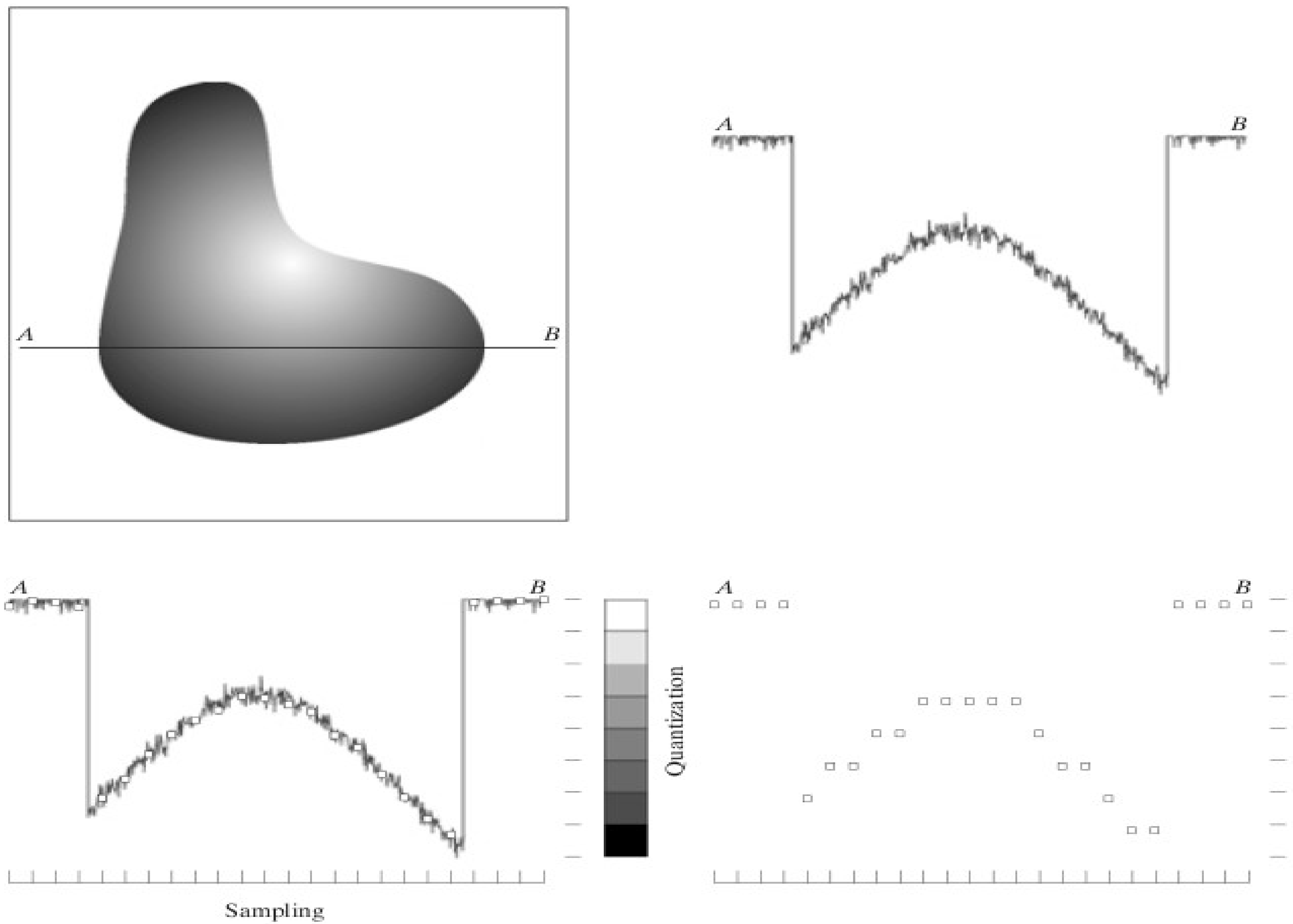
Image Sampling and Quantization



real image | sampled | quantized | sampled & quantized

Objective is to generate digital images from sensed data for, output of most sensors is a continuous voltage waveform

Basic Concepts in Sampling and Quantization



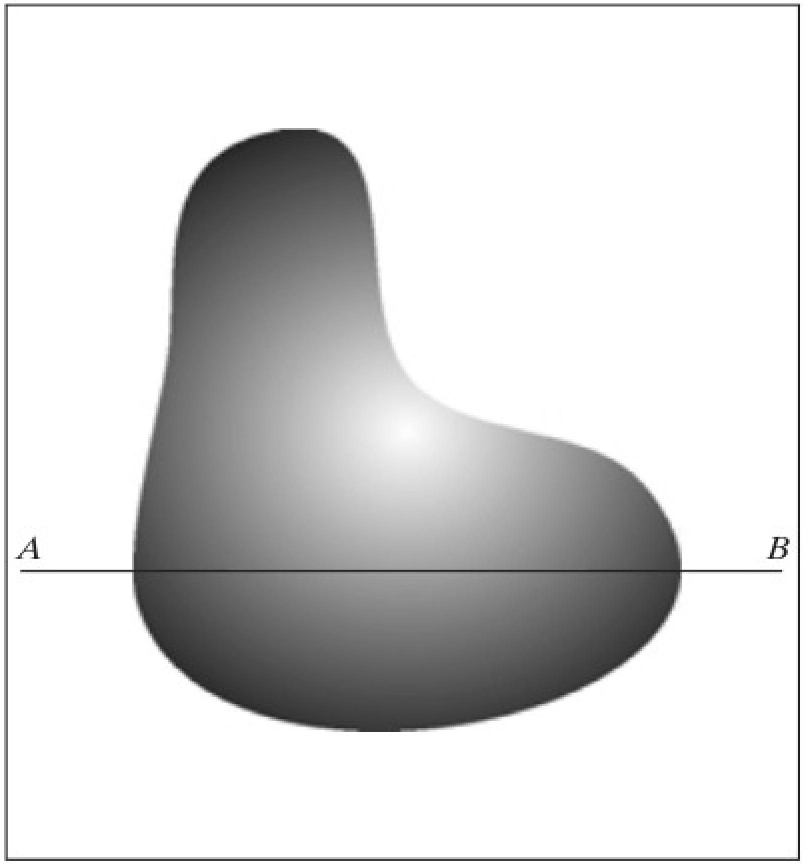
http://ece756.wikispaces.com/

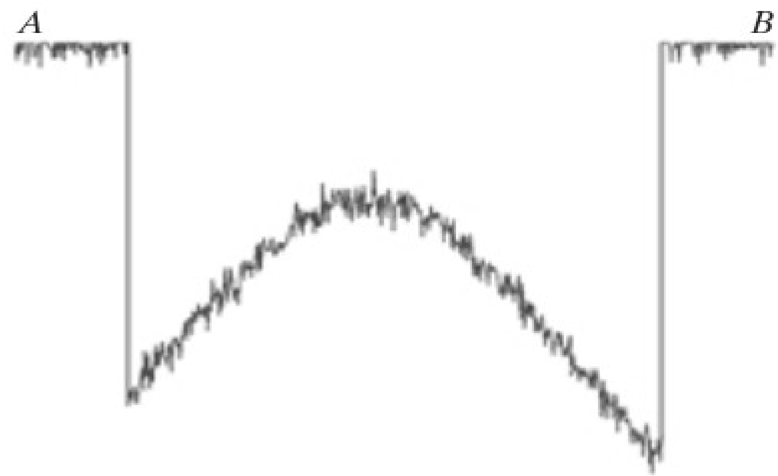
#octave

Sampling and Quantization

- An image may be continuous with respect to the x- and y-coordinates,and also in amplitude.
- To convert it to digital form, we have to sample the function in both coordinates and in amplitude.
- Digitizing the coordinate values is called ***sampling.***
- Digitizing the amplitude values is called ***quantization***.

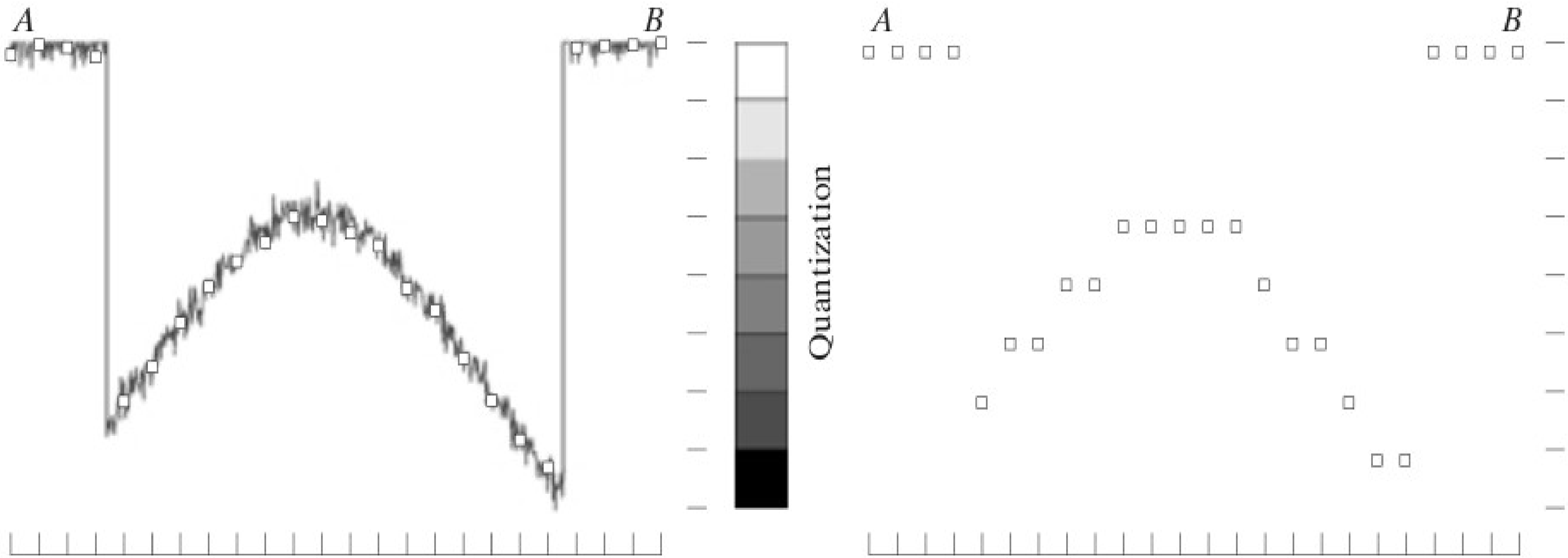
Sampling and Quantization





The one-dimensional function shown in the figure above is a plot of amplitude (gray level) values of the continuous image along the line segment AB. The random variations are due to image noise.

Sampling and Quantization

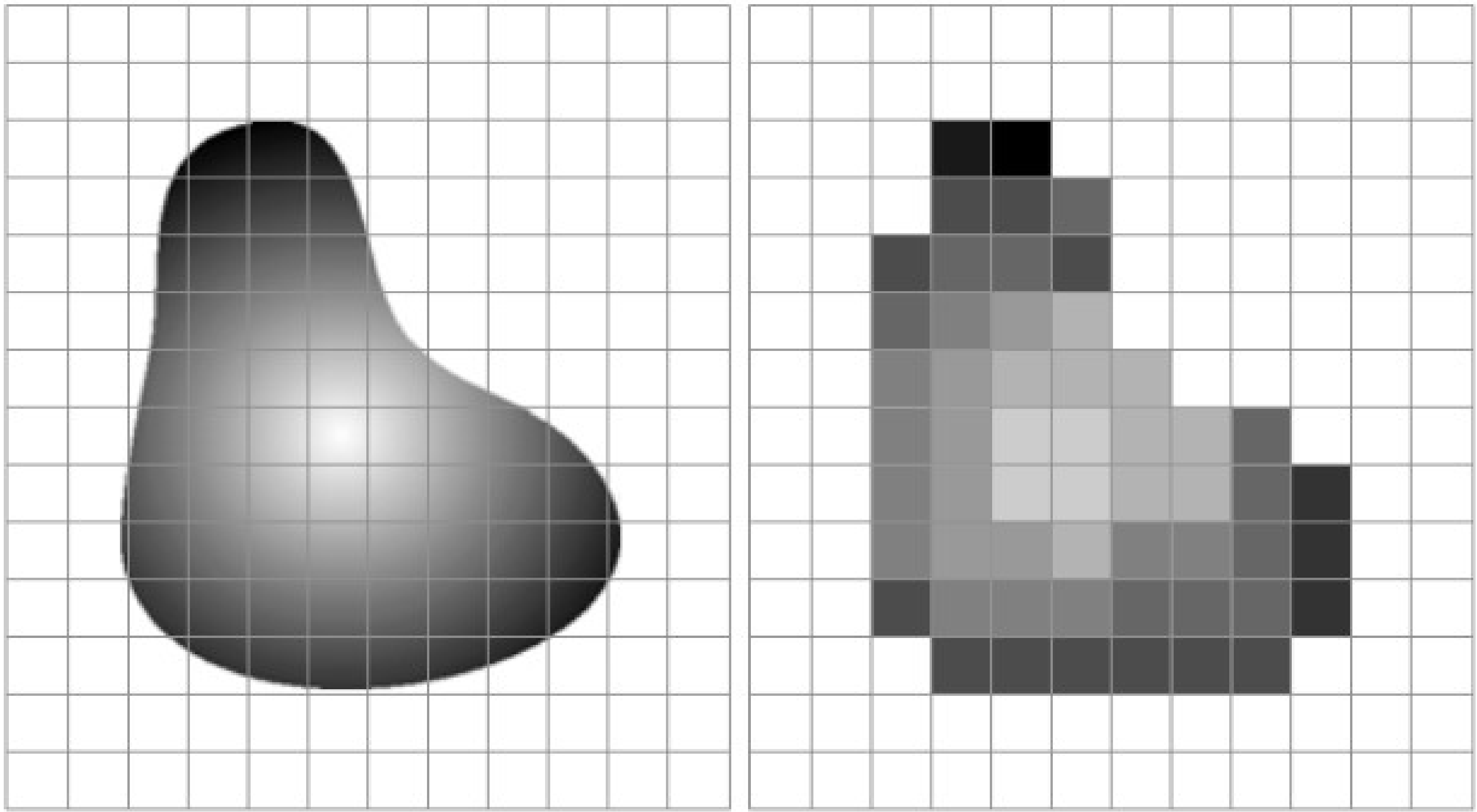


The values of the samples still span (vertically) a continuous range of gray-level values. In order to form a digital function, the gray-level values also must be converted (quantized) into discrete quantities

Sampling and Quantization in relation to the different methods of image acquisition

- *Single sensor* coupled with linear mechanical motion sensing : sampling is accomplished by selecting the number of individual mechanical increments at which we activate the sensor to collect data
- When a *sensing strip* is used for image acquisition, the number of sensors in the strip establishes the sampling limitations in one image direction. Mechanical motion in the other direction can be controlled more accurately

Sampling and Quantization in relation to the different methods of image acquisition



• When a sensing array is used for image acquisition, there is no motion and the number of sensors in the array establishes the limits of sampling in both directions.

Representing Digital Images

```
octave:14> x=imread('gnu.jpg')
x =

  255  255  251  255  252  251  219  203  206  228  253  255  254  255  253  255  255  250  255  255  249  255  255  249  238  243  248  255  252  255  254  252
  250  251  255  222  167  182  167  178  137  139  155  208  249  244  255  255  253  255  252  253  255  237  175  168  163  175  188  179  230  254  245  255
  255  253  195  174  183   53   73  128  157  140  187  209  133  224  250  253  251  254  255  255  156  191  191  139  128  105   63  139  184  191  255  251
  255  191  186  119   16  170  165  184  208  199  181  190  204  255  255  254  255  254  255  249  231  185  192  188  201  180  174   75   60  186  213  255
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  195  240   69   69  178  252  248  255  253  253  209  182  218  173  175  178  203  176  180  182  190  205  195  251  254  255  251  191  137   44  218  209
  183  237   55   97  176  255  253  255  255  212  186  113   10    0    6   52  169  110  108   30    3   51  183  202  255  246  255  254  135   22  222  189
  175  245   37  114  203  253  255  251  219  173   80    3    0    3    0    3   79  251  181  158   22    2   33  171  214  255  252  255  146   21  231  170
  182  236   56  118  180  255  255  230  170   95    1    4   14   32   16   82  134  252  226  178   69    4    1   61  173  240  254  242  143   31  227  177
  189  239   64   68  168  226  219  161  115   51  144  180  234  206  221  255  227  183  176  228  247  167  192   70   90  178  218  177  134   38  242  167
  194  223  113    4  142  181  155   72  113  243  255  240  135   42  233  250  166  214  186  195   87  187  235  245   60   61  162  164    7   79  216  207
  251  176  184   16    4   34   22   80  255  254  247  176  213  200  222  198  219  199  247  211  121  220  220  252  201    7    6    6   16  168  171  244
  253  197  203  175   43    8    0  208  249  255  250  245  249  215  118   77  183  221  246  228   85   59  212  255  255   48   18   48  177  198  216  255
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  254  249  255  254  226  157  255  219  163   87  248  255  247  251  255  253  140  150  201  230  182  227  247  220  164  250  255  255  255  250  254  255
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  250  255  255  252  255  253  229  152  255  236  160  255  255  254  254  214  186  255  255  253  255  250  189  121  248  255  255  255  255  252  255  255
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  254  255  255  255  255  249  252  129  255  251  253  111  223  255  250  251  239  165  164  171  166  129   95  136  253  247  255  255  250  255  255  255
  255  249  255  254  252  255  255  162  227  255  250  145  221  246  255  254  255  249  243  246  248  255  203  226  255  249  255  250  255  255  255  253
  255  255  253  253  255  255  249  241  163  252  255  235  121  129  253  255  255  242  182  125   98  131  163  247  255  255  255  255  255  255  255  255
  255  255  255  254  255  255  254  250  194  206  250  255  237  105  153  202  248  255  209  207  130  252  245  255  255  255  255  255  255  255  255  255
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  255  255  255  255  255  254  253  253  255  254  255  252  250  234  111  193   18  150   17   10  220  252  255  255  255  255  255  255  255  255  255  255
  255  254  254  255  255  255  255  254  255  252  252  254  255  238  213  169   38   40    4    0   38  140  241  250  255  255  255  255  255  255  255  255
  255  253  254  255  255  255  255  255  251  255  254  254  255  251  255  223  191   38   33   62   27  187  243  255  255  255  255  255  255  255  255  255
  255  254  254  255  255  253  254  255  255  253  255  255  255  255  250  255  252  237  234  229  244  255  255  250  255  255  255  255  255  255  255  255
```
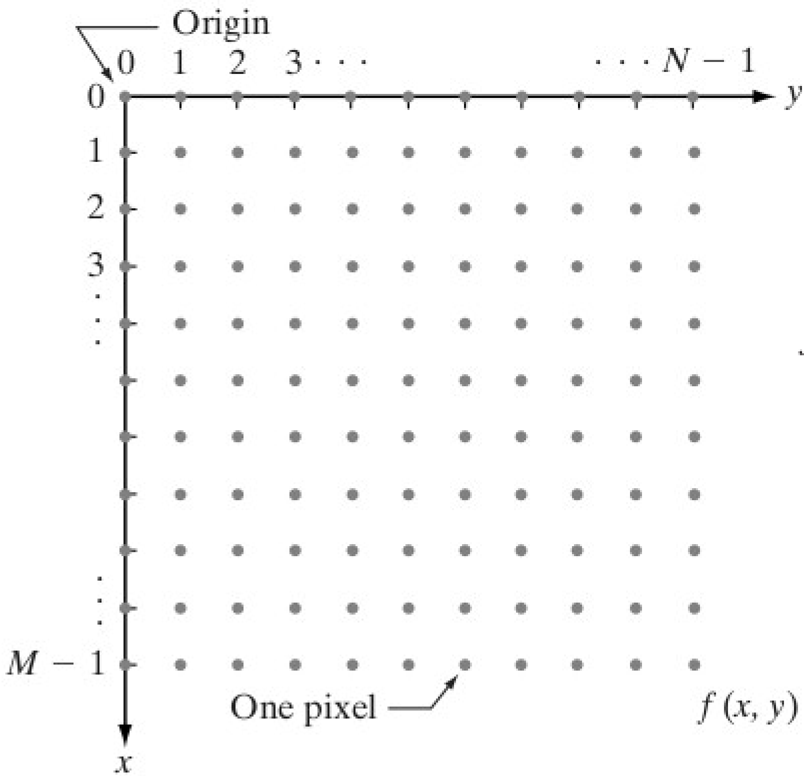

• The result of sampling and quantization is a matrix of real numbers.

Representing Digital Images: Different notations



$$f(x, y) = \begin{bmatrix} f(0,0) & f(0,1) & \cdots & f(0, N-1) \\ f(1,0) & f(1,1) & \cdots & f(1, N-1) \\ \vdots & \vdots & & \vdots \\ f(M-1,0) & f(M-1,1) & \cdots & f(M-1, N-1) \end{bmatrix}.$$

$$\mathbf{A} = \begin{bmatrix} a_{0,0} & a_{0,1} & \cdots & a_{0,N-1} \\ a_{1,0} & a_{1,1} & \cdots & a_{1,N-1} \\ \vdots & \vdots & & \vdots \\ a_{M-1,0} & a_{M-1,1} & \cdots & a_{M-1,N-1} \end{bmatrix}.$$

Also can be denoted in more formal mathematical notations using **Z** and **R**

Representing Digital Images: Different notations

The number, b, of bits required to store a digitized image is

$$b = M \times N \times k.$$

When $M = N$, this equation becomes

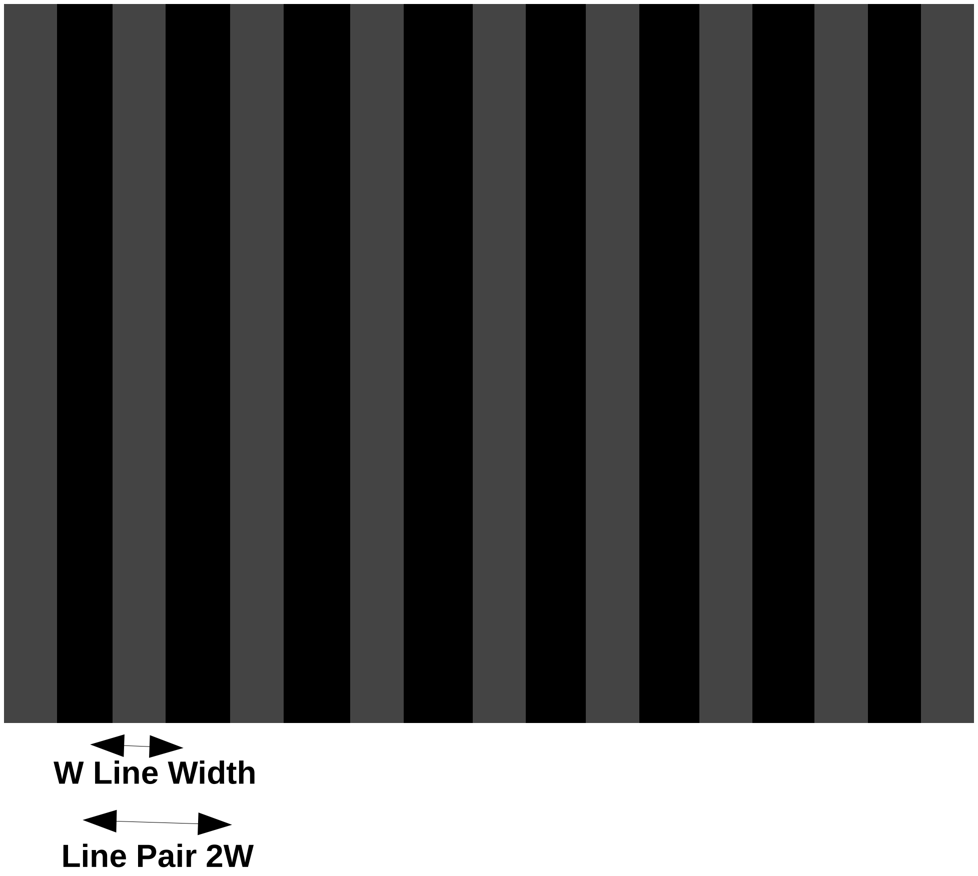
$$b = N^2 k.$$

Number of storage bits for various values of N and k.

N/k	1 ($L = 2$)	2 ($L = 4$)	3 ($L = 8$)	4 ($L = 16$)	5 ($L = 32$)	6 ($L = 64$)	7 ($L = 128$)	8 ($L = 256$)
32	1,024	2,048	3,072	4,096	5,120	6,144	7,168	8,192
64	4,096	8,192	12,288	16,384	20,480	24,576	28,672	32,768
128	16,384	32,768	49,152	65,536	81,920	98,304	114,688	131,072
256	65,536	131,072	196,608	262,144	327,680	393,216	458,752	524,288
512	262,144	524,288	786,432	1,048,576	1,310,720	1,572,864	1,835,008	2,097,152
1024	1,048,576	2,097,152	3,145,728	4,194,304	5,242,880	6,291,456	7,340,032	8,388,608
2048	4,194,304	8,388,608	12,582,912	16,777,216	20,971,520	25,165,824	29,369,128	33,554,432
4096	16,777,216	33,554,432	50,331,648	67,108,864	83,886,080	100,663,296	117,440,512	134,217,728
8192	67,108,864	134,217,728	201,326,592	268,435,456	335,544,320	402,653,184	469,762,048	536,870,912

Spatial and Gray-Level Resolution

- ***Spatial resolution*** is the smallest discernible detail in an image
- *Sampling* is the principal factor determining the spatial resolution of an image
- Resolution is the smallest number of discernible ***line pairs*** per unit distance
- Ex: 100 line pairs per millimeter



Spatial and Gray-Level Resolution

- ***Gray-level resolution*** refers to the smallest discernible change in gray level (which btw is highly subjective)
- Considerable discretion can be accomplished regarding the number of samples used to generate a digital image, but this is not true for the number of gray levels
- The most common number is 8 bits, with 16 bits being used in some applications where enhancement of specific gray-level ranges is necessary.

Subsampling of a Gray-scale image

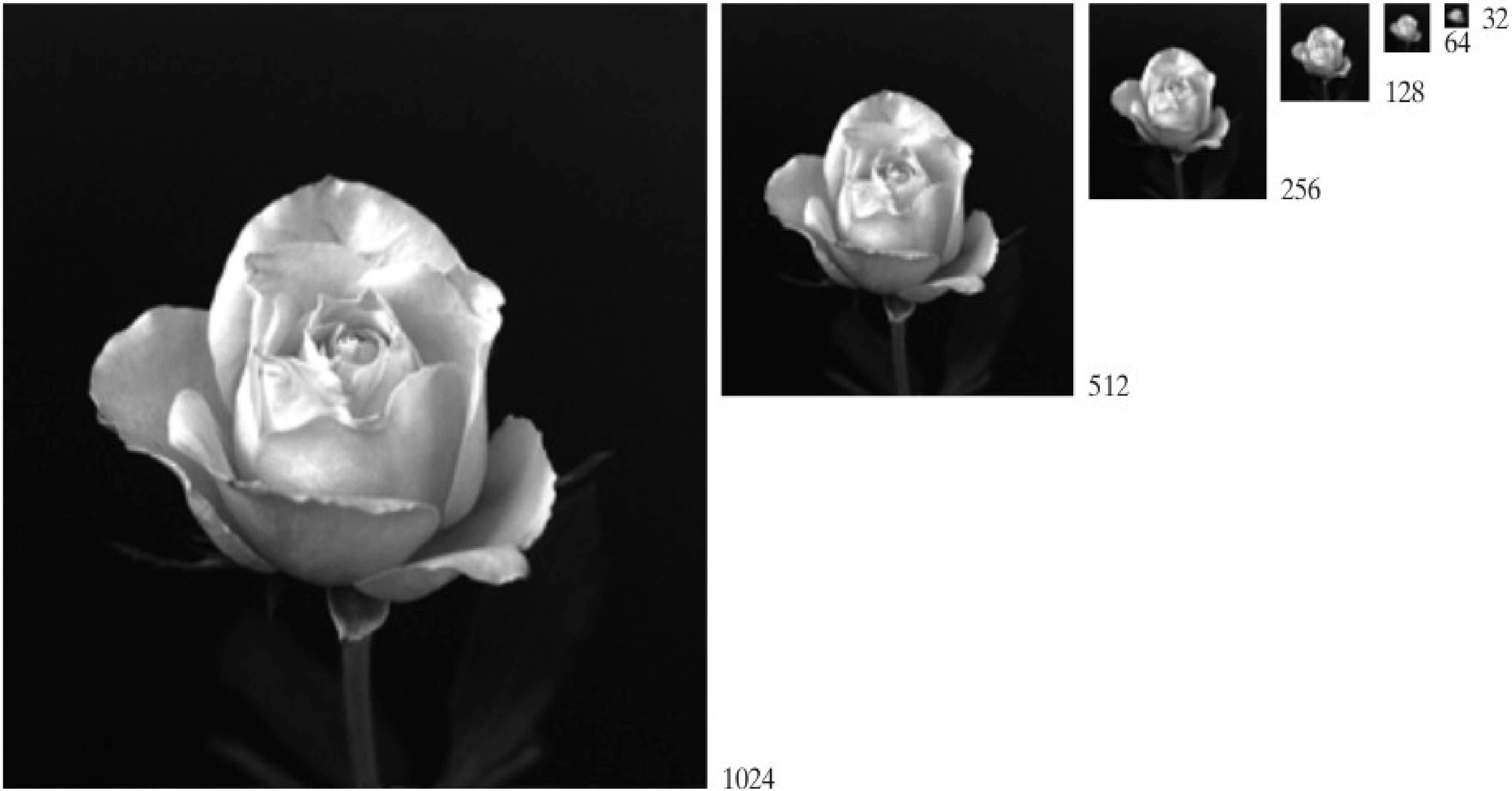
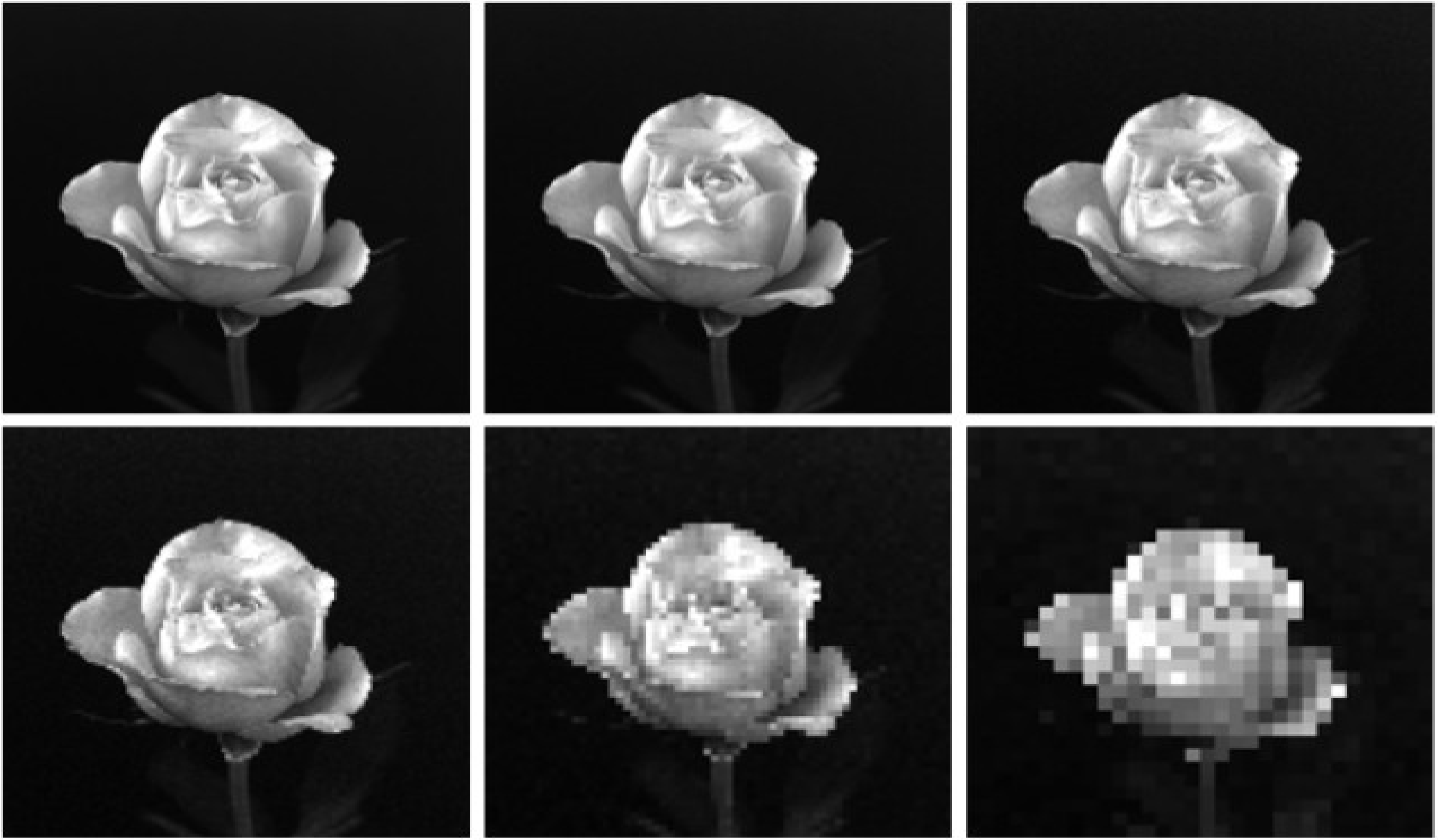


FIGURE 2.19 A 1024 × 1024, 8-bit image subsampled down to size 32 × 32 pixels. The number of allowable gray levels was kept at 256.

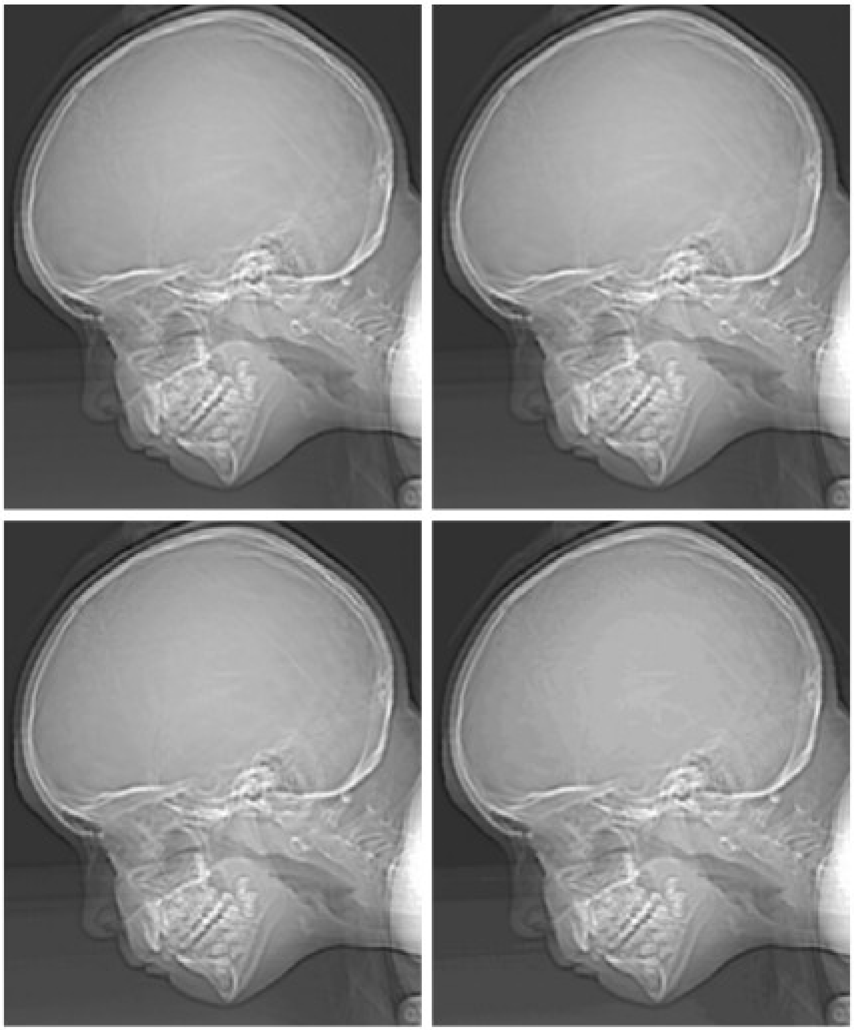
Resampling of a Gray-scale image



a	b	c
d	e	f

FIGURE 2.20 (a) 1024×1024, 8-bit image. (b) 512×512 image resampled into 1024×1024 pixels by row and column duplication. (c) through (f) 256×256, 128×128, 64×64, and 32×32 images resampled into 1024×1024 pixels.

Typical effects of varying the number of gray levels in a digital image.

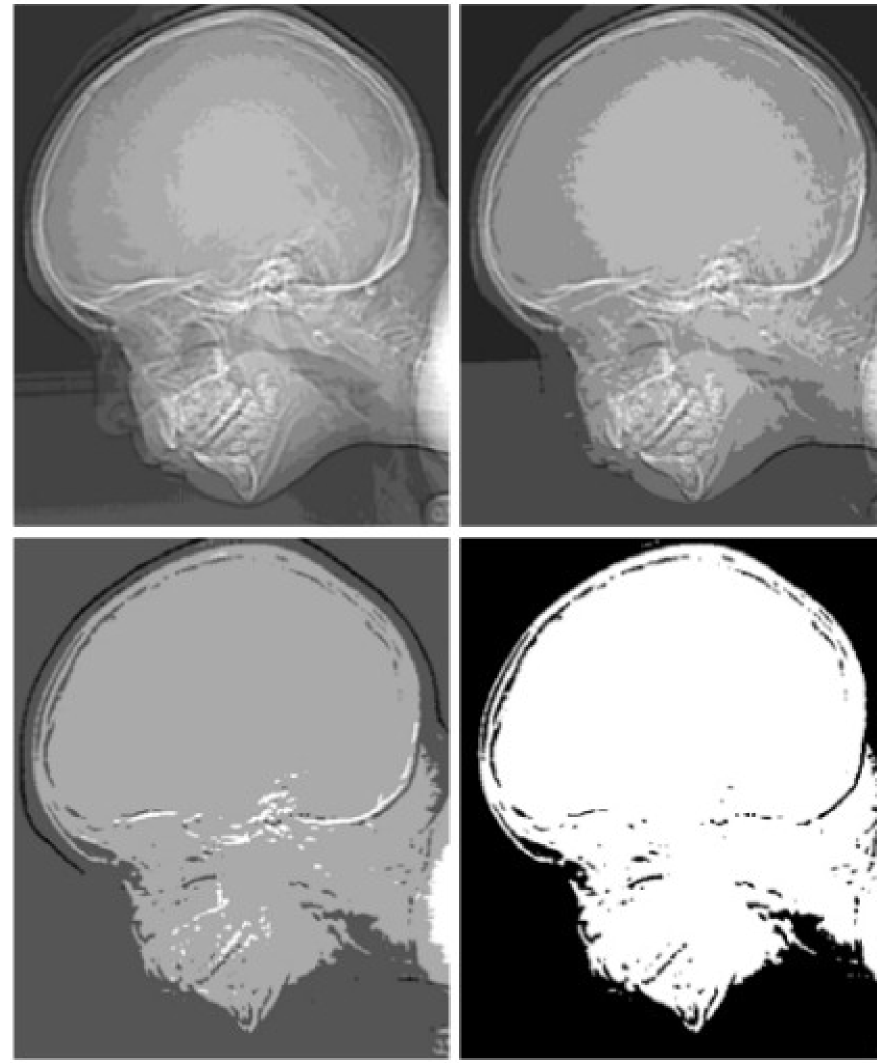


a b
c d

FIGURE 2.21
(a) 452 × 374, 256-level image. (b)–(d) Image displayed in 128, 64, and 32 gray levels, while keeping the spatial resolution constant.

e f
g h

FIGURE 2.21 *(Continued)*
(e)–(g) Image displayed in 16, 8, 4, and 2 gray levels. (Original courtesy of Dr. David R. Pickens, Department of Radiology & Radiological Sciences, Vanderbilt University Medical Center.)



Crude rule of thumb for sampling and quantization

*Assuming powers of 2 for convenience,images of size 256*256 pixels and 64 gray levels are about the smallest images that can be expected to be reasonably free of objectionable sampling checkerboards and false contouring*

Effects produced on image quality by varying N and k independently and simultaneously

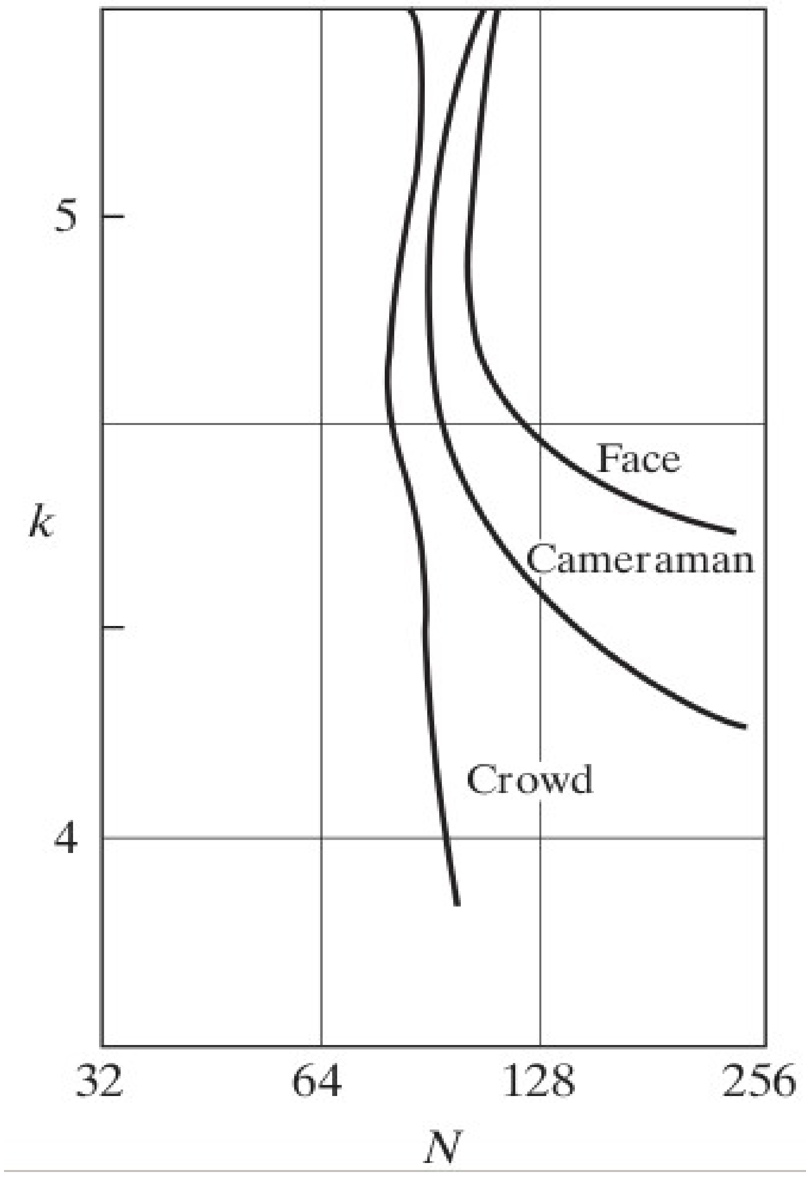
(a) Image with a low level of detail. (b) Image with a medium level of detail. (c) Image with a relatively large amount of detail

Isopreference curves

- An early study by Huang [1965] attempted to quantify experimentally the effects on image quality produced by varying N and k simultaneously
- Three sets of images : With little, intermediate and large amount of detail
- Sets of these three types of images were generated by varying N and k
- Observers were then asked to rank them according to their subjective quality.

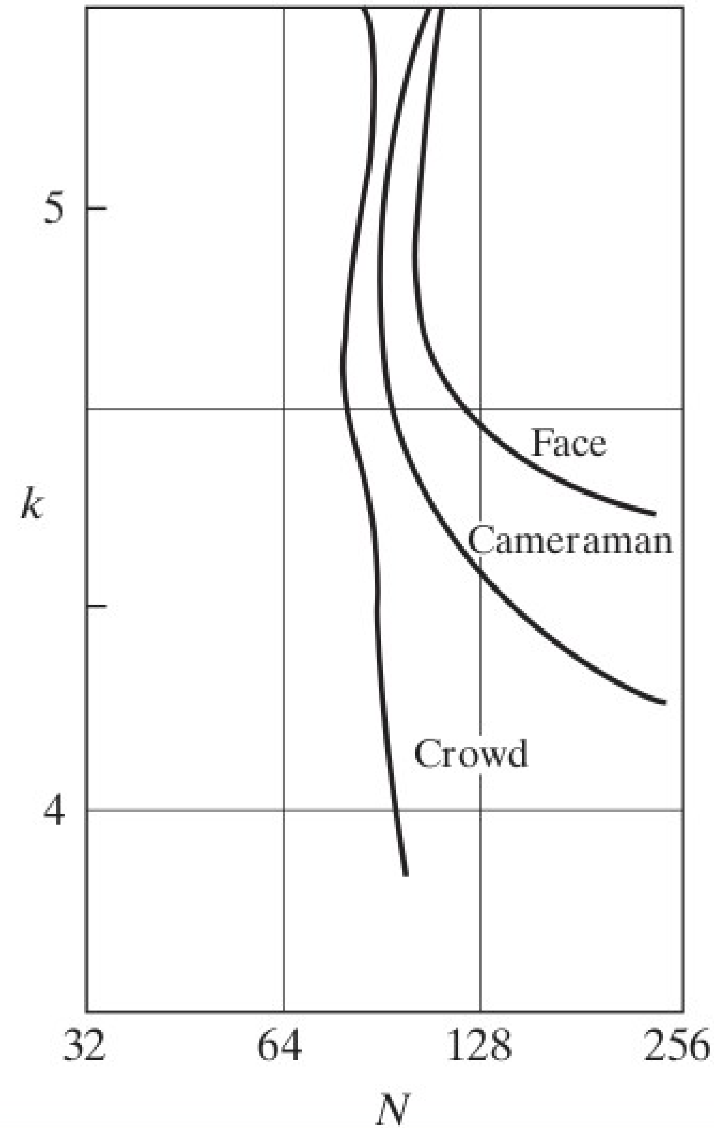
Isopreference curves

• Each point in the Nk-plane represents an image having values of N and k equal to the coordinates of that point
• Points lying on an isopreference curve correspond to images of equal subjective
 quality
• It was found that the *isopreference curves* tended to shift *right* and *upward*, but their shapes in each of the three image categories were *similar*



Inferences from the Isopreference curves

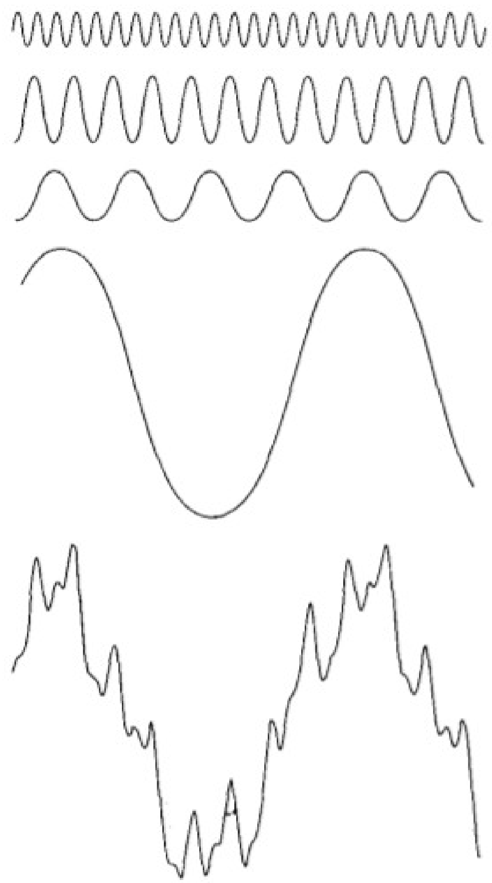
• A shift up and right in the curves simply means larger values for N and k, which implies ***better picture quality.***
• Curves tend to become more *vertical* as the *detail in the image increases*.
• For images with a large amount of detail only a few gray levels may be needed
• This indicates that, for a fixed value of N, the perceived quality for this type of image is nearly independent of the number of gray levels used
• Perceived quality in the other two image categories remained the same in some intervals in which the spatial resolution was increased, but the number of gray levels actually decreased.
•A decrease in k tends to increase the apparent contrast of an image, a visual effect that humans often perceive as improved quality in an image.



Fourier Series and Fourier Transforms

$$f(x) = \frac{a_0}{2} \quad + \quad a_1 \cos(x) + a_2 \cos(2x) + a_3 \cos(3x) + \ldots + a_n \cos(nx)$$
$$\quad + \quad b_1 \sin(x) + b_2 \sin(2x) + b_3 \sin(3x) + \ldots + b_n \sin(nx)$$

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux} \, dx$$



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Fourier Series and Fourier Transforms

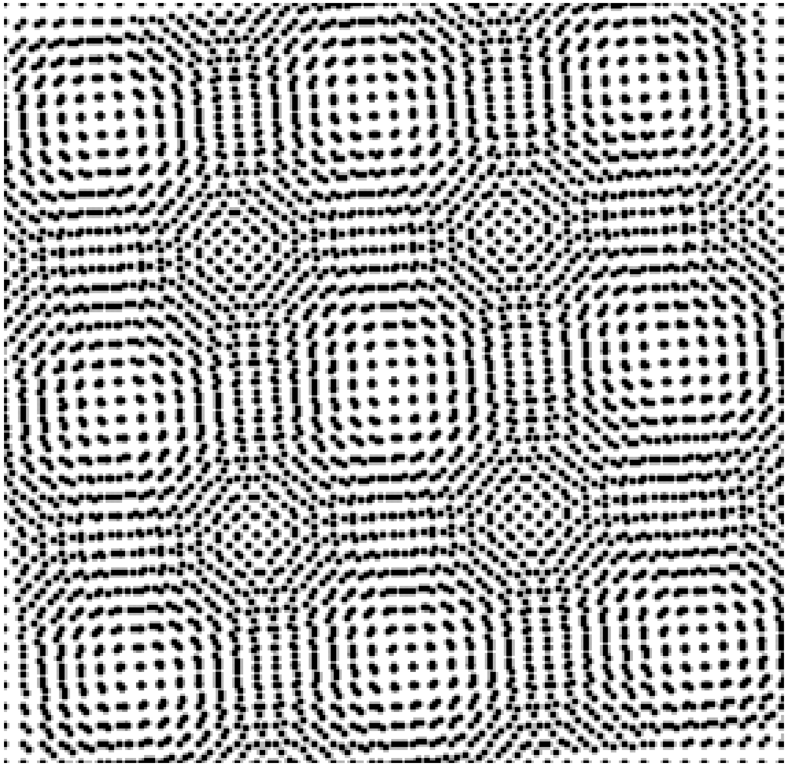
For any periodic function *f(x)*

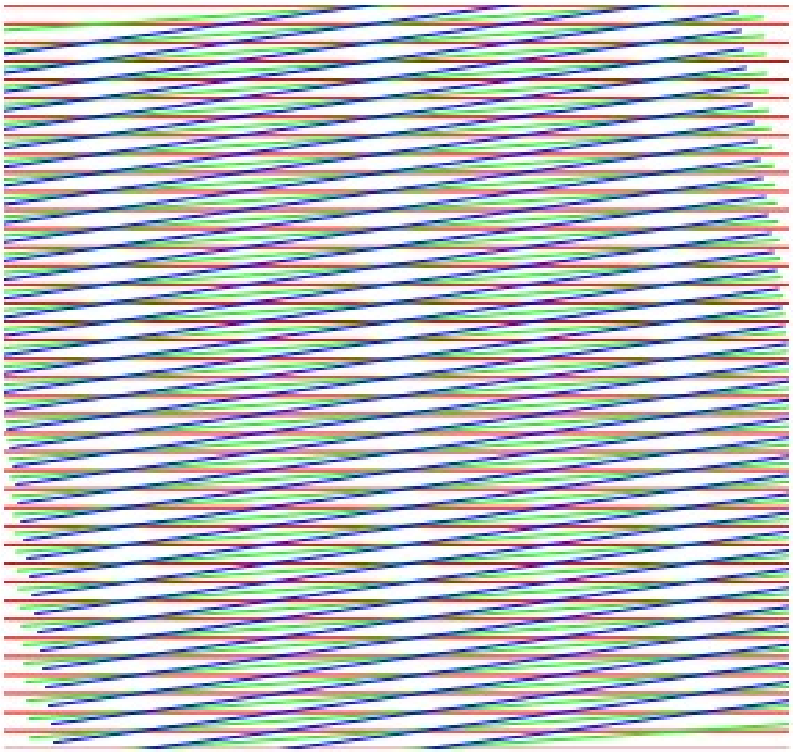
$$f(x) = \frac{a_0}{2} \quad + \quad a_1 \cos(x) + a_2 \cos(2x) + a_3 \cos(3x) + \ldots + a_n \cos(nx)$$
$$+ \quad b_1 \sin(x) + b_2 \sin(2x) + b_3 \sin(3x) + \ldots + b_n \sin(nx)$$

For any non-periodic function *f(x)*

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux} \, dx$$

Aliasing and Moiré* Patterns





*say mwahr

Aliasing and Moiré* Patterns

• Functions whose area under the curve is finite can be represented in terms of sines and cosines of various frequencies.
• The sine/cosine component with the highest frequency determines the highest "frequency content" of the function.
• If the function is band-limited, then Shannon's Sampling theorem can be applied
• If the function is *undersampled*, then a phenomenon called *aliasing* corrupts the sampled image
• The corruption is in the form of *additional frequency* components being introduced into the sampled function called *Aliased frequencies*

Note: *The sampling rate in images is the number of samples taken (in both spatial directions) per unit distance.*

•

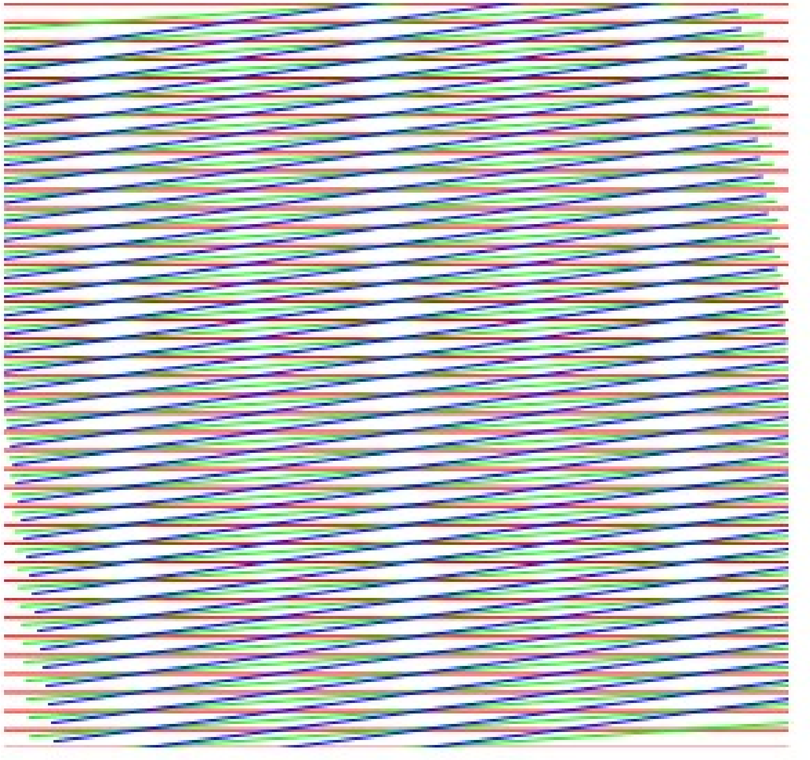
*say mwahr

Practical constraints of sampling/ Paradox of sampling

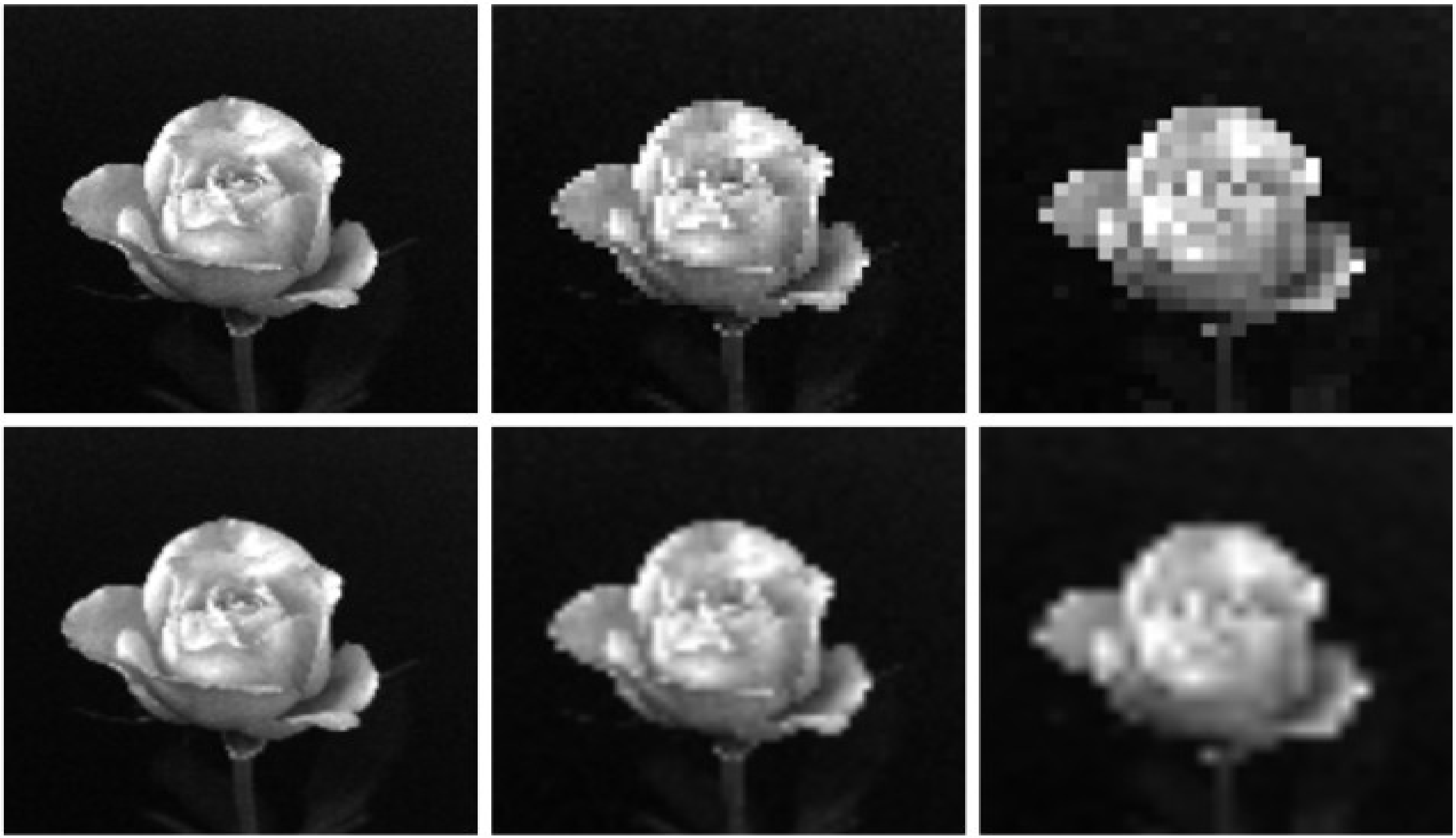
• In functions of infinite duration (hence finite highest frequencies) taking infinite samples is not feasible (but for one exception)
• We can work only with *finite samples of data*
• A function of unlimited duration is processed into a function of finite duration simply by multiplying the unlimited function by a “gating function” (||||lr to the Step functions)
• The gating function itself has frequency components that extend to infinity
• *The very act of limiting the duration of a band-limited function causes it to cease being band limited*
• Reducing the aliasing effects on an image is to reduce its high-frequency components by blurring the image *prior to sampling*
• The effect of *aliased frequencies* under the right conditions form the **Moiré Patterns**

What are Moiré* Patterns?

• **Special case:** a function of *infinite duration* can be sampled over a *finite interval* without violating the sampling theorem.
•With $\mathbf{f_s \geq 2f_h}$ **,** it is possible to recover the function from its samples provided that the sampling captures *exactly an integer number of periods* of the function.
• The Moiré pattern in the figure is caused by a breakup of the periodicity between the three overlapping images



Zooming and Shrinking Digital images



a b c
d e f

FIGURE 2.25 Top row: images zoomed from 128 × 128, 64 × 64, and 32 × 32 pixels to 1024 × 1024 pixels, using nearest neighbor gray-level interpolation. Bottom row: same sequence, but using bilinear interpolation.

Zooming and Shrinking Digital images

- Zooming -> Oversampling
- Shrinking -> Undersampling
- Key difference between Zooming and Shrinking, Sampling and Quantization
- Continuous and Digital images

Steps involved in Zooming

- Creation of new pixel locations
- Assignment of gray levels to those new locations.
- In order to perform gray-level assignment for any point in the overlay (newly inserted pixels), we look for the closest pixel in the original image and assign its gray level to the new pixel in the grid

Methods for zooming

140x140
Original
a=imread('gnu.jpg')

- **Nearest Neighbour Interpolation**
- **Pixel Replication (special case of Nearest neighbour intr.)**
- ➢ **Bilinear Interpolation**

210x210
Nearest neighbour Interpolation
imresize(a,1.5,'nearest')

210x210
Pixel replication
imresize(a,1.5)

210x210
Bilinear interpolation
imresize(a,1.5,'bilinear')

Methods for zooming: Nearest Neighbour Interpolation

- ➢ Lay an overlay (grid) of the desired increased size of the image
- ➢ The spacing in the grid would be less than one pixel because we are fitting it over a smaller image
- ➢ Gray-level assignments to be made for any point in the overlay (newly inserted pixels)
- ➢ Look for the closest pixel in the original image and assign its gray level to the new pixel in the grid.
- • When we are done with all points in the overlay grid, we simply expand it to the original specified size to obtain the zoomed image.
- ➢ Undesirable feature: *Checkerboard effect,* particularly objectionable at high magnification

Methods for zooming: Pixel Replication

- Special case of nearest neighbor interpolation
- Pixel replication is applicable when we want to increase the size of an image an integer number of times
- Duplication of rows and columns is done the required number of times to achieve the desired size
- The gray-level assignment of each pixel is predetermined by the fact that new locations are exact duplicates of old location

Methods for zooming: Bilinear Interpolation

- Overcomes the checkerboard effect from previous methods
- Uses four nearest neighbours of a point

Let ***(x', y')*** denote the coordinates of a point in the zoomed image (think of it as a point on the grid described previously), and let ***v(x', y')*** denote the gray level assigned to it.
For bilinear interpolation, the assigned gray level is given by

$$v(x', y') = ax' + by' + cx'y' + d$$

where the four coefficients are determined from the four equations in four unknowns that can be written using the four nearest neighbors of point

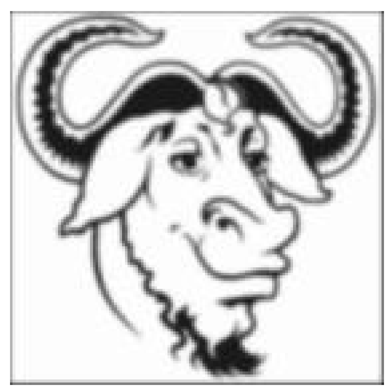
Shrinking of digital images

- Image shrinking is performed in a similar manner as zooming
- The equivalent process of pixel replication is row-column deletion. For example, to shrink an image by one-half, we delete every other row and column.
- We can use the zooming grid analogy to visualize the concept of shrinking by a noninteger factor, except that we now expand the grid to fit over the original image, do gray-level nearest neighbor or bilinear interpolation, and then shrink the grid back to its original specified size.
- To reduce possible aliasing effects, it is a good idea to blur an image slightly before shrinking it

More on zooming and shrinking

• It is possible to use more neighbors for interpolation.
• Using more neighbors implies fitting the points with a more complex surface, which generally gives smoother results.This is an exceptionally important consideration in image generation for 3-D graphics and in medical image processing
• But the extra computational burden seldom is justifiable for general-purpose digital image zooming and shrinking, where bilinear interpolation generally is the method of choice.

210x210
Bilinear interpolation
imresize(a,1.5,'bilinear')



140x140
Original
a=imread('gnu.jpg')

210x210
Bicubic interpolation
imresize(a,1.5,'bicubic')

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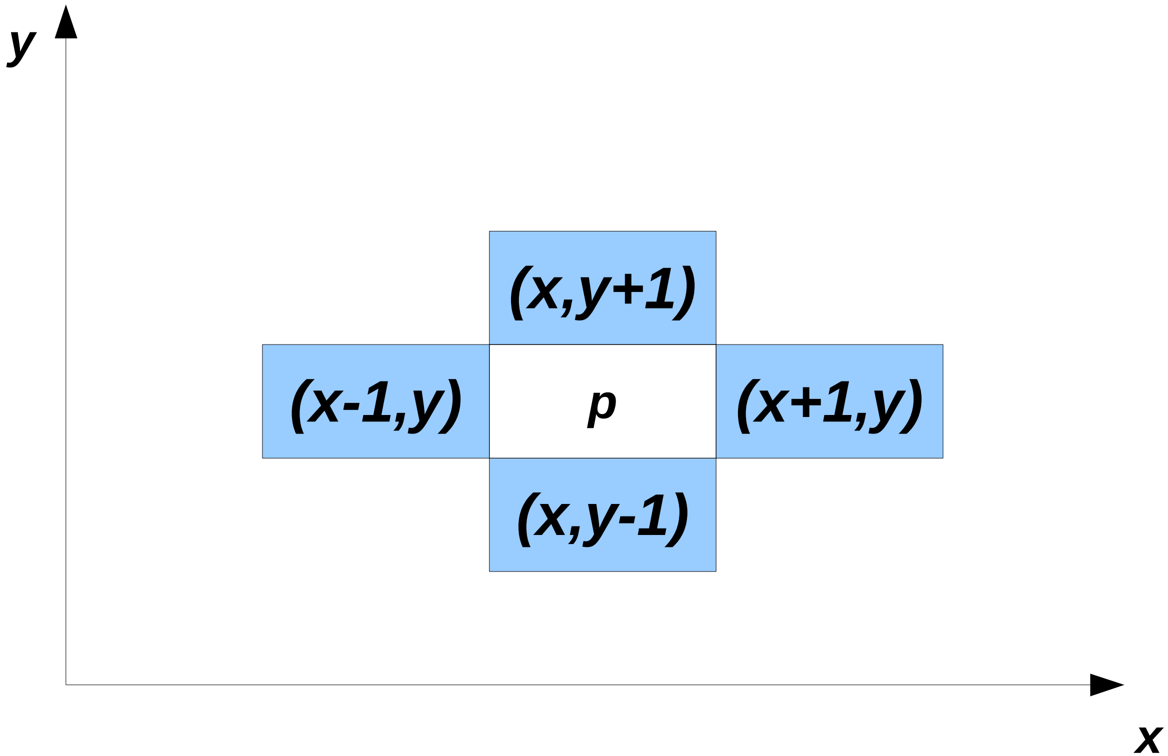
Basic Relationships Between Pixels

- Neighbors of a Pixel
- Adjacency, Connectivity, Regions, and Boundaries
- Distance Measures
- Image Operations on a Pixel Basis
- Linear and Nonlinear Operations

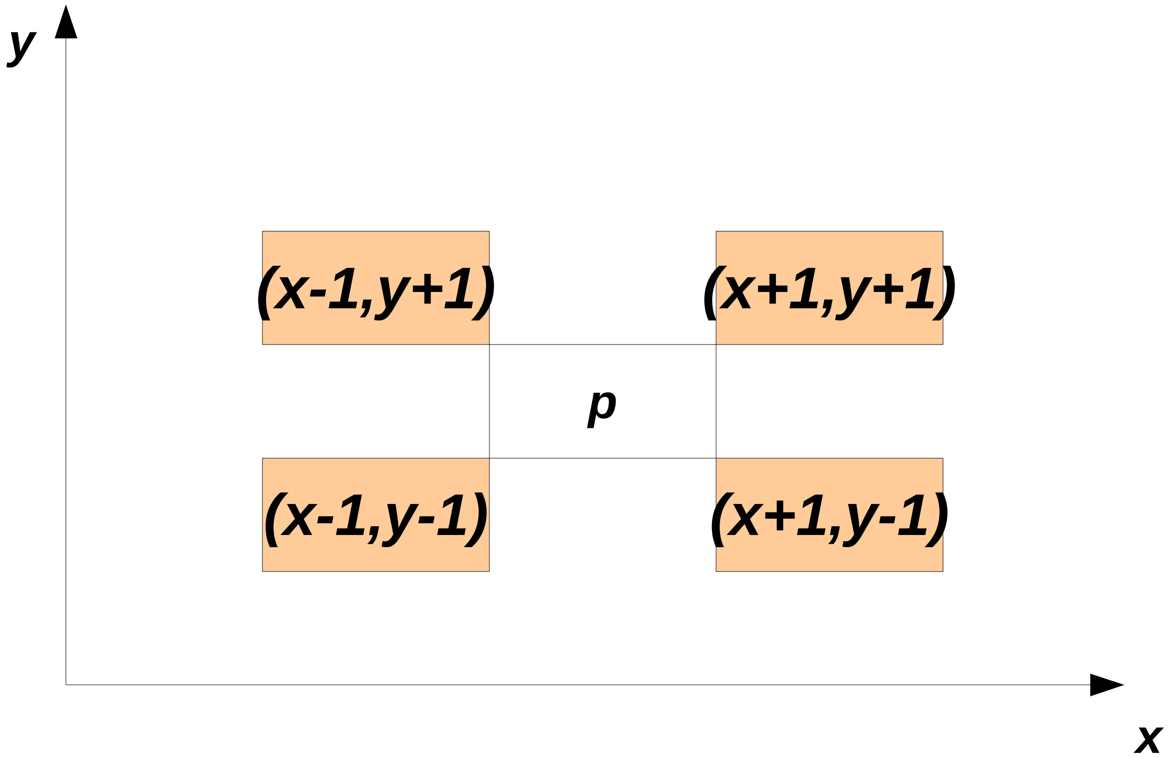
4 Neighbours of a Pixels: $N_4(p)$



A pixel p at coordinates (x, y) has four horizontal and vertical neighbors whose coordinates are given by
(x+1, y), (x-1, y), (x, y+1), (x, y-1)

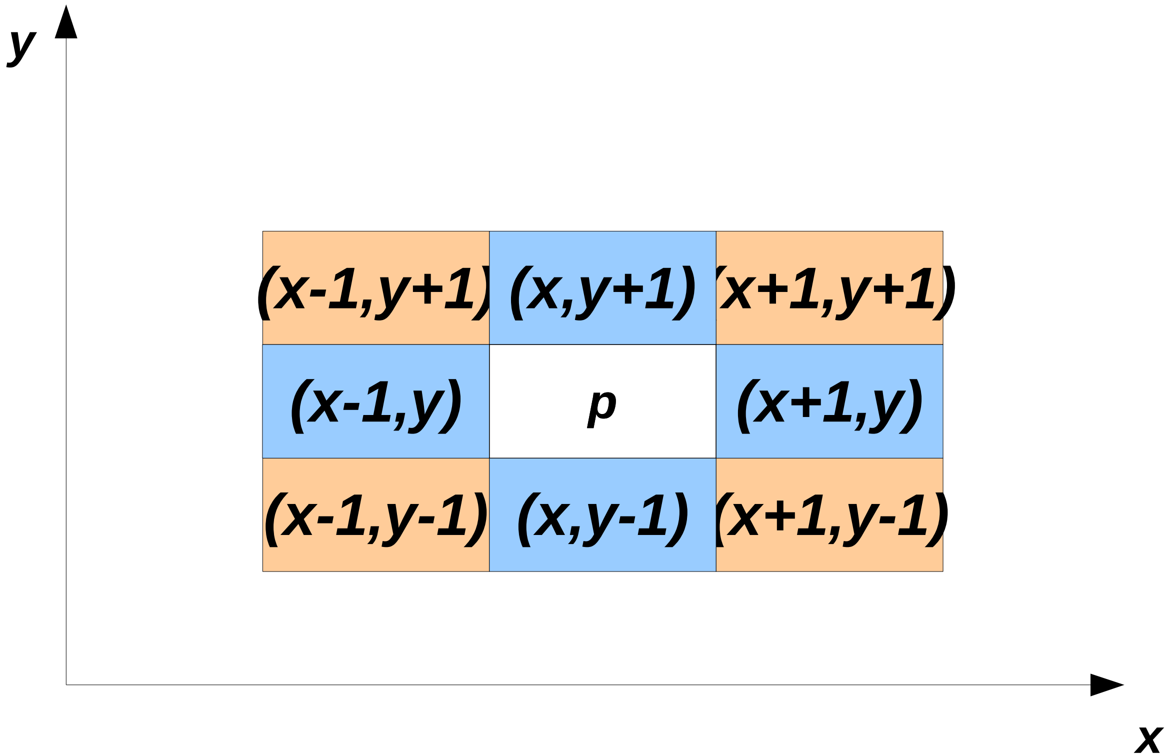
Some of the neighbors of p lie outside the digital image if (x, y) is on the border of the image.

4 Neighbours of a Pixels: Diagonal Neigbours-$N_D(p)$



The four diagonal neighbors of p have coordinates
(x+1, y+1), (x+1, y-1), (x-1, y+1), (x-1, y-1)

8 Neighbours of a Pixels: $N_8(p)$



Adjacency, Connectivity, Regions, and Boundaries

- Concept of Connectivity between pixels simplifies digital image concepts such as regions and boundaries
- To *establish if two pixels are connected*, it must be determined if they are *neighbors* and if their *gray levels satisfy* a specified criterion of similarity
- ***V*** will be used to denote the set of gray-level values used to define adjacency

  Ex: For a binary image ***V***={0,1},and so on

Types of pixel adjacency

1) ***4-adjacency:*** Two pixels p and q with values from ***V*** are 4-adjacent if q is in the set $\mathbf{N_4(p)}$

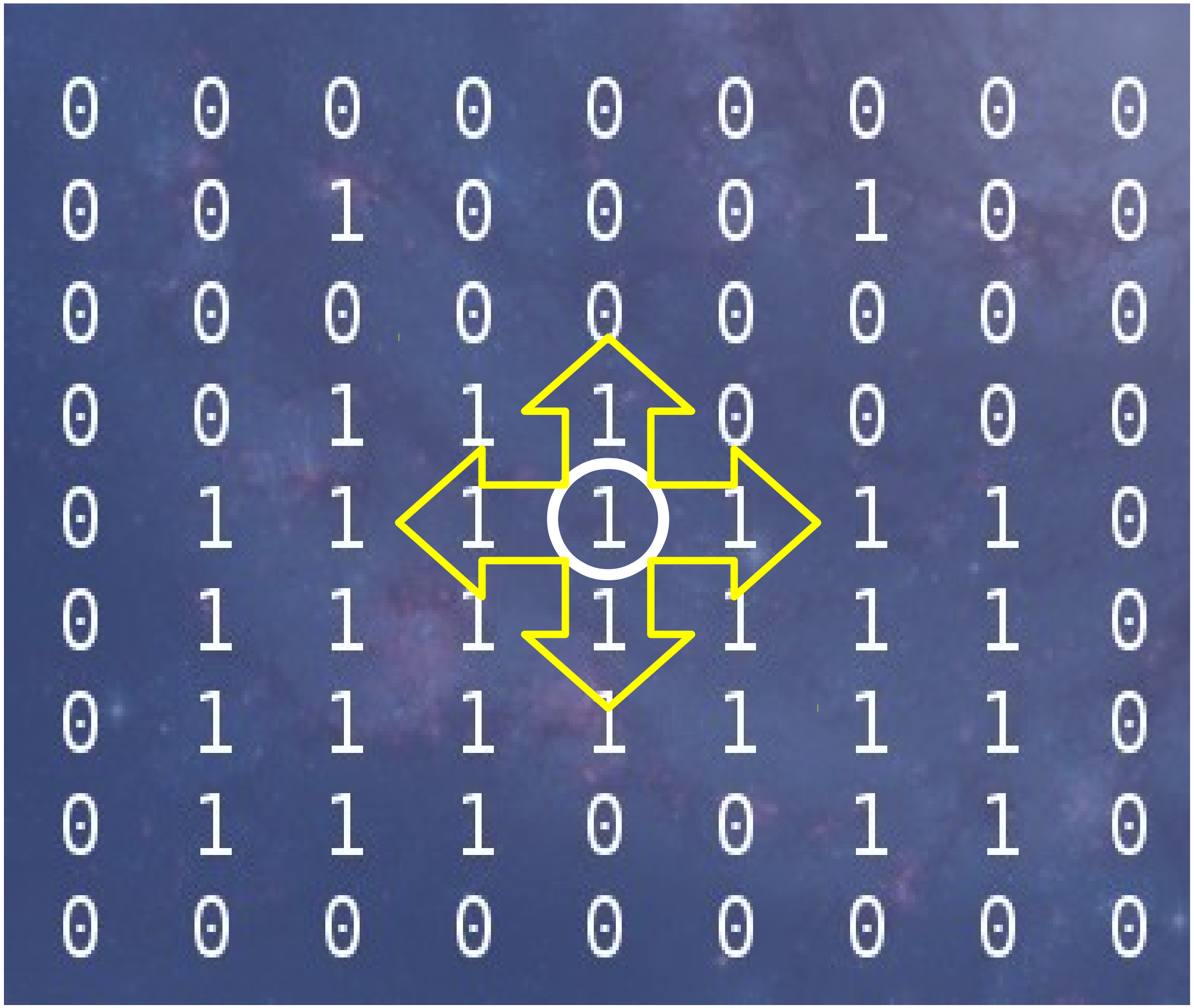
2) ***8-adjacency:*** Two pixels p and q with values from ***V*** are 8-adjacent if q is in the set $\mathbf{N_8(p)}$

3) ***m-adjacency (mixed adjacency):*** Two pixels p and q with values from ***V*** are m-adjacent if

(i) q is in $\mathbf{N_4(p)}$, or

(ii) q is in $\mathbf{N_D(p)}$ and the set "$\mathbf{N_4(p) \cap N_4(q)}$" has no pixels whose values are from ***V***

4-neighbours $N_4(p)$



V={0,1}

8-neighbours $N_8(p)$

V={0,1}

m-adjacency

	2		
1	P 1	2	
	0		

V={0,1}

N_4(p)={0,1,2,2}

		0	
	2	Q 1	1
		2	

V={0,1}

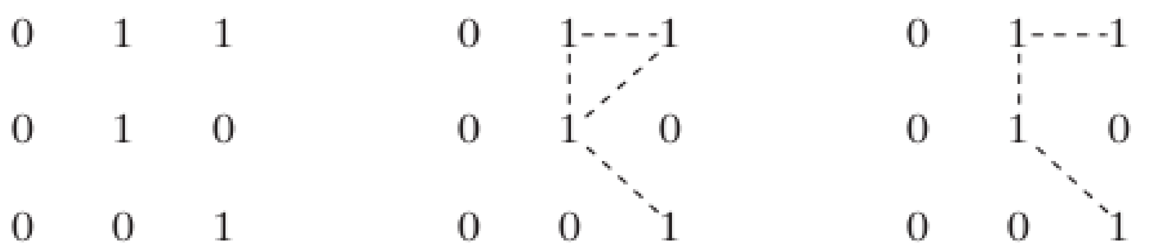
N_4(q)={2,2,0,1}

		0	
	2	Q 1	1
1	P 1	2	
	0		

V={0,1}

N_4(p) ∩ N_4(q)= {2,2) !∑ V

Types of pixel adjacency



(a) Arrangement of pixels; (b) pixels that are 8-adjacent (shown dashed) to the center pixel; (c) m-adjacency

- **Mixed adjacency is a modification of 8-adjacency.**
- **It is introduced to eliminate the ambiguities that often arise when 8-adjacency is used**

Image Subset adjacency

- **Two image subsets S1 and S2 are adjacent if some pixel in S1 is adjacent to some pixel in S2.**

Digital path and pixel connectedness

Digital Path*:* A (digital) path (or curve) from pixel p with coordinates (x, y) to pixel q with coordinates (s, t) is a sequence of distinct pixels with coordinates

$$(x_0 , y_0),(x_1 , y_1),........,(x_n , y_n)$$

Where $(x_0 , y_0)=(x,y); (x_n , y_n)= (s,t)$

And pixels (x_i , y_i) *and* (x_{i-1} , y_{i-1}) *are adjacent for* $1 \leq i \leq n$

"n" is the length of the path

If $(x_0 , y_0)=(x_n , y_n)$, then the path is a ***closed path***

$$\begin{array}{ccccl} 0 & 1 & 1 & & \text{p: } (x_0 , y_0)=(x,y) \\ 0 & 1 & 0 & & \\ 0 & 0 & 1 & & \text{q: } (x_3 , y_3)=(s,t) \end{array}$$

Connected Set “S”

Let ***S*** represent a subset of pixels in an image.

Two pixels p and q are said to be connected in S if there exists a *path between them* consisting *entirely* of pixels in S.

For any pixel p in ***S***, the set of pixels that are connected to it in ***S*** is called a *connected component* of ***S***.

If it only has one connected component, then set S is called a ***connected set***.

Boundary of a region R

Let ***R*** be a subset of pixels in an image.

We call ***R*** a *region of the image,* if ***R*** is a ***connected set***.

The boundary (also called border or contour) of a region ***R***
is the set of pixels in the region that have one or more neighbors
that are not in ***R***.

If ***R*** happens to be an entire image (which we recall is a rectangular set of pixels), then its boundary is defined as the set of pixels in the first and last rows and columns of the image.

Edges and Boundaries

The **boundary** of a finite region forms a closed path and is thus a “global” concept.

Edges are formed from pixels with derivative values that exceed a preset threshold. The idea of an edge is a “local” concept that is based on a measure of gray-level discontinuity at a point

Edges are intensity discontinuities and boundaries are closed paths.

Distance Measures

For pixels p, q, and z, with coordinates (x, y), (s, t), and (v, w), respectively, D is a distance function or metric if

(a) *D(p, q) ≥ 0 (D(p, q)=0 iff p=q)*

(b) *D(p, q)=D(q, p)* and

(c) D(p, z) ≤ D(p, q)+D(q, z)

The Euclidean distance between p and q is defined as

$$D_e(p, q) = \left[(x - s)^2 + (y - t)^2\right]^{\frac{1}{2}}.$$

For this distance measure, the pixels having a distance less than or equal to some value ***r*** from ***(x, y)*** are the points contained in a disk of radius ***r*** centered at ***(x, y)***

Distance Measures

The *D4 distance* (also called *city-block distance*) between p and q is defined as

$$\mathbf{D_4(p, q) = |x - s| + |y - t|}$$

In this case, the pixels having a $\mathbf{D_4}$ distance from ***(x, y)*** less than or equal to some value r form a *diamond* centered at (x, y). For example, the pixels with $\mathbf{D_4}$ distance ≤ 2 from (x, y) (the center point) form the following contours of constant distance:

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

The pixels with D_4=1 are the 4-neighbors of (x, y)

Distance Measures

The D_8 *distance* between p and q is defined as

$$\mathbf{D_8(p, q) = max(|x - s| + |y - t|)}$$

In this case, the pixels with $\mathbf{D_8}$ distance from ***(x, y)*** less than or equal to some value r form a square centered at ***(x, y)***

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

The pixels with D_8=1 are the 8-neighbors of (x, y)

Note: The D_4 and D_8 distances between p and q are independent of any paths that might exist between the points because these distances involve only the coordinates of the points

Distance Measures

The D_m *distance* between two points is defined as the shortest m-path between the points.

	p_3	p_4
p_1	p_2	
p		

Image Operations on a Pixel Basis

Arithmetic operations on Pixel Matrix :Add/Sub/Logical

Division?

Linear and Nonlinear Operations

Let H be an operator whose input and output are images.
H is said to be a ***linear operator*** if, for any two images f and g and any two scalars a and b,

$$\mathbf{H(af + bg) = aH(f) + bH(g).}$$

An operator that fails the test of above Eq is by definition ***nonlinear***.